



An Integrated Spherical Fuzzy Multi-criterion Group Decision-Making Approach and Its Application in Digital Marketing Technology Assessment

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Abstract

Digital marketing technology can not only help enterprises effectively control marketing costs but also accurately identify target customers and determine customer needs to improve marketing conversion rates. Multiple conflicting and asymmetric criteria need to be taken simultaneously into account to effectively assess digital marketing technology. Accordingly, the target of our paper is to propound an enhanced additive ratio assessment (ARAS) methodology to aid experts in evaluating digital marketing technology in a complex uncertain setting. The spherical fuzzy (SF) set is an expansion of the picture fuzzy set (PFS) that has been recognized as a powerful model to portray indeterminacy and vagueness. A fresh score function is proffered to compare the SF numbers and its merits are also analyzed. Then, some novel operations for SF numbers are defined and several aggregation operators based on them are propounded. Further, a mixed multi-criteria group decision-making (MCGDM) method is developed by incorporating the method based on the removal effects of criteria (MERECE), step-wise weight assessment ratio analysis (SWARA), and enhanced ARAS approach under SF information. Also, a case concerning digital marketing technology evaluation is executed to prove the efficiency and practicability of the presented SF-MERECE–SWARA–ARAS method. Finally, contrast and sensitivity investigations are also implemented to highlight the strengths and robustness of the proffered methodology, respectively.

Keywords Digital marketing technology · SF set · MCGDM · Information fusion · MERECE · ARAS · SWARA

Abbreviations

ARAS	Additive ratio assessment	CRITIC	Criterion importance through inter-criteria correlation
SF	Spherical fuzzy	FMEA	Failure modes and effects analysis
PFS	Picture fuzzy set	SFN	Spherical fuzzy number
MCGDM	Multi-criteria group decision-making	TSFWA	Tangent spherical fuzzy weighted averaging
MERECE	Method based on the removal effects of criteria	TSFWG	Tangent spherical fuzzy weighted geometric
SWARA	Step-wise weight assessment ratio analysis	TSFOWA	Tangent spherical fuzzy ordered weighted averaging
MCDM	Multi-criteria decision-making	TSFOWG	Tangent spherical fuzzy ordered weighted geometric
PROMETHEE	Preference ranking organization method for enrichment evaluations	FSF-DM	Fused spherical fuzzy decision matrix
CRADIS	Compromise ranking of alternatives from distance to ideal solution	CoCoSo	Combined compromise solution
CPT-TODIM	Cumulative prospect theory an acronym in Portuguese for interactive and multi-criteria decision-making	WSM	Weighted sum model
		WASPAS	Weighted aggregated sum product assessment
		COPRAS	Complex proportional assessment

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1 Introduction

The remarkable development of digital technology has made digitalization piecemeal popular in all aspects of human life. As a new marketing model, digital marketing is a way to further improve the efficiency of marketing drawing support from computer communication technology and digital interactive media [1–3]. Digital marketing can not only quickly open new markets for companies but also tap into the needs of consumers at a deeper level. However, in the face of the many intelligent technologies available on the Internet, it is vital to choose the most satisfactory digital marketing technology for the development of a company's digital marketing model [4–7]. Therefore, digital marketing technology evaluation is crucial for enterprises to improve their core competitiveness in the era of data empowerment.

To evaluate digital marketing technology, we need to consider a set of valuable Internet technologies concerning diverse criteria. Digital marketing technology evaluation is regarded as a typical multi-criteria decision-making (MCDM) problem that thinks over the criteria of several assaults and dissymmetric simultaneously. However, with the increasing sophistication of the decision environment, it is hard for experts to identify an ideal digital marketing technology based on criteria from different properties and various rates. To treat the aforementioned problem with uncertainty, a lot of theories such as fuzzy sets [8], intuitionistic fuzzy sets [9], rough sets [10], and soft sets [11] are put forward to deal with this kind of MCDM problems. In addition, several novel extension theories like interval-valued IFS [12], PFS [13], and SF set [14] are propounded to apply in diverse domains [15–17].

As an innovative information representation model, SF set modes the uncertainty from three perspectives, including positive, neutral, and negative membership degrees, and provides a larger range for experts to express opinions. Recently, the theory of SF set has been viewed as a worthy manner to portray impreciseness and vagueness in various MCDM problems [18–20]. Because of its efficiency and extensive practicality, many researchers and scholars have set up diverse decision methodologies and lots of theory research, such as information fusion, information measures, and score functions [21–24]. Apart from these results, Chinram et al. [25] proposed several novel spherical fuzzy Yager operators and constructed a decision algorithm based on them to select the optimal wind power plant location. Gundogdu and Kahraman [26] combined SF set and quality function deployment to deal with the problem of linear delta robot technology design and evaluation. Akram et al. [27] introduced a novel PROMETHEE-based outranking approach under a spherical

fuzzy context. Wang et al. [28] propounded an SF sort algorithm via regret theory for the application of moving high-tech manufacturing. Wang et al. [29] suggest a fresh Fine–Kinney risk assessment framework based on complex SFSs and the CRADIS approach for the appraisal of natural gas pipeline construction. Further, Zhang et al. [30] suggested a CPT-TODIM method to portray the psychological behavior of experts during the MCDM process. They computed weights using the CRITIC method within SF settings. More research can be referred to in the literature [31–35].

In recent years, different types of decision methodologies have been widely applied to practical decision analysis under diverse situations. However, the solution to practical decision problems still presents certain challenges due to factors, such as the complexity of the decision environment and human perception. Criteria and their importance are observed as indispensable concepts for experts to provide reliable assessment information in the MCDM process [36]. In most cases, it is assumed that criterion weights are given directly by the assessor or are considered equally important, which leads to inconsistent decision results. In the existing literature, the determination of criterion weights is usually divided into subjective weights, objective weights, and composite weights [37]. Recently, Keshavarz-Ghorabae et al. [38] proposed a fresh objective weight determination called the MEREC method, which determined the importance of a criterion by measuring the absolute deviation between the assessed value of each criterion removed from the schemes and the assessed value of all criteria. The core idea is that the higher the weight of a criterion, the greater its impact when removed in all cases. Since then, the MEREC has been successively applied to different fuzzy environments to determine the importance of criteria [39]. Furthermore, the SWARA method is a powerful weight-determination method that determines the subjective weight of a criterion by comparing the importance coefficients between the criteria [40]. It has also been adopted to solve various complex decision analyses in different uncertain environments due to its simplicity and ease of operation, as well as its ability to better manifest the subjective appraise of experts on the criteria [41, 42]. At present, there has been no research to merge the MEREC and SWARA methods in the SFS setting. Hence, the union of MEREC and SWARA models is employed for weight identification in this study.

Recent investigations show that many classical decision algorithms are applied or extended to spherical fuzzy circumstances for finding an optimal (satisfied) scheme effectively in realistic decision problems. The above different decision methods are suitable for solving different complicated situations and have different advantages and disadvantages during decision analysis. In addition, the

additive ratio assessment (ARAS) method was originated by Zavadskas and Turskis in 2010 [43]. ARAS is observed as an effective model for tackling complex decision analysis. The core aim of the ARAS model is to select the optimal option from a collection of schemes based on a number of criteria. The ultimate priority of candidates is ascertained by working out the utility degree of each candidate [44]. Based on the existing research about ARAS methodologies, it can be concluded that the ARAS method possesses the following merits: (1) it can deal with complicated decision problems with high complexity; (2) it is a compensatory approach to directly obtain the final utility ratio between the weighted assessment and the optimal evaluation; (3) it has the advantages of the simple calculation process, easy operation, and high robustness for solving MCDM problems. Owing to these merits in dealing with MCDM problems, a multitude of achievements has been developed by scholars to strengthen the efficiency and fidelity of ARAS methodology in different uncertain circumstances to address diverse problems [45–48]. Besides, Jin et al. [49] presented an expanded ARAS method based on FMEA under interval-valued q -rung orthopair fuzzy circumstances for developing risk analysis. Mishra et al. [50] construct a novel single-valued neutrosophic ARAS methodology via a novel similarity measure for the selection of electric vehicle charging stations. Mishra et al. [51] introduced an integrated assessment framework with the aid of a novel Pythagorean fuzzy similarity measure and ARAS model for evaluating the sustainable biomass crop. To model the cognition of experts by linguistic model, Teng and Shen [52] proposed a new MCGDM method based on the SWARA and ARAS approaches with unbalanced double hierarchy linguistic information. Further, the ARAS approach was applied to different applications to highlight its superiority in decision analysis [53–55].

With the help of the mentioned discussions and literature overview investigations, it is evident that SFSs are regarded as an efficient, powerful, and energetic tool to treat the impreciseness and ambiguity in the aspect of information representation. The emergence of the SF set can provide more advantageous methods to support the solution of uncertain decision problem analysis. Thus, the intention for this paper can be concluded as follows:

- SF set-based decision methodology offers an effective model to deal with decision analysis by enlarging the range of three elements as compared to PFS. Therefore, it is crucial to focus on the construction of decision models within the SF set setting.
- The score function is an essential theory for building uncertain decision methodology. Unfortunately, the extant score functions of the SF set possess some defects, which shall cause unreasonable results in

making decisions. To conquer the above shortcomings, it is necessary to explore a novel score function to improve the reasonability when comparing spherical fuzzy numbers (SFNs).

- The combined weights of assessment criteria can think about both objective influences from decision data and subjective judgments from expert knowledge and experience, so that computing the combined weights of the criteria can effectively improve the quality of decisions.
- As a simple and easy-operate decision algorithm, the ARAS method prioritizes a set of schemes based on the utility degree theory. Thus, the combination of ARAS and SF sets produces a stronger MCDM approach than the extant ones.

By means of the mentioned discussions and motivations, the target of this study is to offer a hybrid SF information-based MCGDM methodology for the assessment of digital marketing technologies. The proposed hybrid methodology is constructed by incorporating the MEREC, SWARA, and enhanced ARAS methods under an SF environment. The involved methods are all improved by the novel score function and aggregation operators. Hence, the presented SF-MEREC–SWARA–ARAS method is an innovative group decision framework for experts to unfold decisions within an uncertain context. Consequently, the novelty and contributions of this paper are displayed as follows:

- A fresh score function is presented to enhance the reasonability of SFN comparison, and the associated properties and strengths of it are also explored and analyzed severally.
- Some novel operations are propounded and four novel aggregation operators, such as tangent spherical fuzzy weighted averaging (TSFWA) and tangent spherical fuzzy weighted geometric (TSFWG) operators, and their ordered weighted definitions are presented.
- A fused criteria-weighted determination model based on the improved SF-MEREC and SF-SWARA methods is built for ascertaining the importance of criteria.
- An enhanced ARAS group decision methodology is presented with the aid of the score function and TSFWA operator to obtain the ranking of schemes.
- The case study which uses the proposed SF-MEREC–SWARA–ARAS approach to assess digital marketing technology is implemented to validate its effectiveness and usefulness. Comparison and sensitivity investigations are also performed to analyze the advantage and robustness of the proffered methodology.

To achieve the mentioned aims and contributions, other sections of the current study are planned as below: we briefly look back at several basic conceptions of SFS in

Sect. 2. Then, a novel spherical fuzzy score function and its superiority analysis are presented in Sect. 3. In Sect. 4, several newly operational laws for SFSs are defined and a series of novel aggregation operators are propounded to fuse the uncertain information. A hybrid MCGDM decision methodology called SF- MEREC–SWARA–ARAS is presented to apply the developed score function and operators in Sect. 5. Then, a practical case of digital marketing technology assessment is given in Sect. 6 to expound on the practicability and feasibility of the proposed approach. In addition, we also compare and analyze the distinctions between different spherical fuzzy methods. Finally, the conclusions are summed up in the end.

2 Preliminaries

This section schematically retrospects some fundamental notions of SFS and score functions.

2.1 SFS

Definition 1 [14, 21]. It is supposed that \wp be a universal set. An SFS Υ on X is expressed as

$$\Upsilon = \{ \langle \hat{\lambda}, \phi_{\Upsilon}(\hat{\lambda}), \varphi_{\Upsilon}(\hat{\lambda}), \kappa_{\Upsilon}(\hat{\lambda}) \rangle | \hat{\lambda} \in \wp \}, \quad (1)$$

wherein $\phi_{\Upsilon}(\hat{\lambda}) : \wp \rightarrow [0, 1], \varphi_{\Upsilon}(\hat{\lambda}) : \wp \rightarrow [0, 1]$ and

$\kappa_{\Upsilon}(\hat{\lambda}) : \wp \rightarrow [0, 1]$ stand for the positive, neutral, and negative membership degrees of the element $\hat{\lambda} \in \wp$ to Υ , which fulfills the condition that $0 \leq (\phi_{\Upsilon}(\hat{\lambda}))^2 + (\varphi_{\Upsilon}(\hat{\lambda}))^2 + (\kappa_{\Upsilon}(\hat{\lambda}))^2 \leq 1$ for all $\hat{\lambda} \in \wp$. The refusal grade is indicated as $\delta_{\Upsilon}(\hat{\lambda}) = \sqrt{1 - ((\phi_{\Upsilon}(\hat{\lambda}))^2 + (\varphi_{\Upsilon}(\hat{\lambda}))^2 + (\kappa_{\Upsilon}(\hat{\lambda}))^2)}$. For the sake of simplicity, $\Upsilon = (\phi, \varphi, \kappa)$ is called SFN.

Definition 2 [21] Suppose that $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t) (t = 1, 2)$, then one has the following:

- (1) $\Upsilon_1 = \Upsilon_2$ iff $\phi_1 = \phi_2, \varphi_1 = \varphi_2, \kappa_1 = \kappa_2$;
- (2) $\Upsilon_1 \cap \Upsilon_2 = (\min\{\phi_1, \phi_2\}, \min\{\varphi_1, \varphi_2\}, \max\{\kappa_1, \kappa_2\})$;
- (3) $\Upsilon_1 \cup \Upsilon_2 = (\max\{\phi_1, \phi_2\}, \max\{\varphi_1, \varphi_2\}, \min\{\kappa_1, \kappa_2\})$;
- (4) $\Upsilon^c = (\phi, \varphi, \kappa)$.

Definition 3 [21]. For two SFNs $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t) (t = 1, 2)$, then the comparison laws defined based on the score function S and accuracy function A are expressed as follows:

- (1) If $S(\Upsilon_1) > S(\Upsilon_2)$, then $\Upsilon_1 \succ \Upsilon_2$
- (2) If $S(\Upsilon_1) = S(\Upsilon_2)$ and $A(\Upsilon_1) < A(\Upsilon_2)$, then $\Upsilon_1 \prec \Upsilon_2$
- (3) If $S(\Upsilon_1) = S(\Upsilon_2)$ and $A(\Upsilon_1) = A(\Upsilon_2)$, then $\Upsilon_1 \sim \Upsilon_2$.

2.2 The Piori Spherical Fuzzy Score Functions

A score function is a crucial tool for comparing the SFNs and obtaining the order relation of SFNs. In this subsection, we first review some existing spherical fuzzy score function in reference [14, 21], and then analyze their defects by some numerical examples. It is supposed that $\Upsilon = (\phi, \varphi, \kappa)$ be an SFN. The extant score functions for SFN are illustrated below:

1. The score and accuracy functions proposed by Mahmood et al. [14] and Ashraf et al. [21]:

$$S_1(\Upsilon) = \phi^2 - \kappa^2,$$

$$H_1(\Upsilon) = \phi^2 + \varphi^2 + \kappa^2.$$

2. The score function defined by Ashraf and Abdullah [22]:

$$S_2(\Upsilon) = \frac{1}{3}(2 + \phi - \varphi - \kappa).$$

3. The score and accuracy functions proposed by Chinram et al. [25]:

$$S_3(\Upsilon) = \frac{1}{3}(2 + \phi^2 - \varphi^2 - \kappa^2).$$

4. The score function proposed by Gündoğdu and Kahraman [24]:

$$S_4(\Upsilon) = (\phi - \kappa)^2 - (\varphi - \kappa)^2.$$

3 A Novel Spherical Fuzzy Score Function

This section first analyzes the shortcomings of the prior score functions and then propounds an innovative score function for the SF set with the help of ponding the refusal grade information.

3.1 Shortcomings Analysis for the Piori Spherical Fuzzy Score Functions

The extant SF score and accuracy functions can compare SFNs and obtain the corresponding order relations. Nevertheless, the mentioned score functions have several shortcomings that obtain the unreasonable ranking in comparing and ranking the SFNs in several cases. Some examples are provided to demonstrate the irrational situation.

Example 1. Let $\Upsilon_1 = (0.3, 0.5, 0.3)$ and $\Upsilon_2 = (0.6, 0.5, 0.6)$ be two SFNs. Then, we can obtain that the calculation outcomes are expressed as $S_1(\Upsilon_1) = S_1(\Upsilon_2)$, $S_2(\Upsilon_1) = S_2(\Upsilon_2)$ and $S_3(\Upsilon_1) = S_3(\Upsilon_2)$, which shows that the score functions proposed in reference [14, 21, 22, 25] fail to distinguish the SFNs Υ_1 and Υ_2 .

Example 2. Let $\Upsilon_1 = (0.6, 0.6, 0)$ and $\Upsilon_2 = (0.5, 0.5, 0)$ be two SFNs. Then, we can acquire that the calculation results are expressed as $S_2(\Upsilon_1) = S_2(\Upsilon_2)$, $S_3(\Upsilon_1) = S_3(\Upsilon_2)$, and $S_4(\Upsilon_1) = S_4(\Upsilon_2)$, which shows that the score function proposed in reference [22, 24, 25] fails to distinguish the SFNs Υ_1 and Υ_2 .

Example 3. Let $\Upsilon_1 = (0.3, 0, 0.3)$ and $\Upsilon_2 = (0.2, 0, 0.2)$ be two SFNs. Then, we can attain that the score values of Υ_1 and Υ_2 are $S_1(\Upsilon_1) = S_1(\Upsilon_2)$, $S_2(\Upsilon_1) = S_2(\Upsilon_2)$ and $S_3(\Upsilon_1) = S_3(\Upsilon_2)$, which means that the score function proposed in reference [14, 21, 22, 25] cannot compare them. Furthermore, we further compare the accuracy value of Υ_1 and Υ_2 displayed as $H_1(\Upsilon_1) > H_1(\Upsilon_2)$, namely, $\Upsilon_1 \succ \Upsilon_2$. While, the order of Υ_1 and Υ_2 determined by score functions S_4 is $\Upsilon_1 \prec \Upsilon_2$, which they obtain the counterintuitive outcomes.

Example 4. Let $\Upsilon_1 = (0.3, 0.2, 0.4)$ and $\Upsilon_2 = (0.3, 0.3, 0.3)$ be two SFNs. Then, we can acquire that the calculation results are expressed as $S_2(\Upsilon_1) = S_2(\Upsilon_2)$ which shows that the score function proposed in reference [22] fails to distinguish the SFNs Υ_1 and Υ_2 .

In light of the aforementioned discussion, we will put forward an innovative spherical fuzzy score function to conquer the deficiencies in the previous works and attain a more reasonable ranking of SFNs.

3.2 New Score Function

This subsection shall propose a novel spherical fuzzy score function and discuss its valuable properties and superiorities.

Definition 4. Given an SFN $\Upsilon = (\phi, \varphi, \kappa)$, a fresh score function $\widehat{S}(\Upsilon)$ is propounded as

$$\widehat{S}(\Upsilon) = \frac{1}{2} \left(1 + \phi^2 - \kappa^2 - (\varphi^2 + \delta^2) \times \frac{\log_2(1 + \varphi^2 + \delta^2)}{100} \right). \quad (2)$$

Theorem 1. The propounded score function $\widehat{S}(\Upsilon)$ of SFN $\Upsilon = (\phi, \varphi, \kappa)$ increases monotonically with ϕ and decreases monotonically with ϕ and κ .

Proof. In view of Eq. (2), we first obtain the partial derivative of the developed score function $\widehat{S}(\Upsilon)$ with ϕ as

$$\frac{\partial \widehat{S}(\Upsilon)}{\partial \phi} = \phi \geq 0.$$

Homoplastically, we also derive the partial derivative of the developed score function $\widehat{S}(\Upsilon)$ with φ and κ as

$$\frac{\partial \widehat{S}(\Upsilon)}{\partial \varphi} = - \left(\varphi \frac{\log_2(1 + \varphi^2 + \delta^2)}{100} + (\varphi^2 + \delta^2) \frac{1}{100(1 + \varphi^2 + \delta^2) \ln 2} \right) \leq 0,$$

$$\frac{\partial \widehat{S}(\Upsilon)}{\partial \kappa} = -\kappa \leq 0.$$

Accordingly, the theorem is proved.

Theorem 2. Given an SFN $\Upsilon = (\phi, \varphi, \kappa)$, the proposed score function $\widehat{S}(\Upsilon)$ obeys the following axioms.

$$(P1) \widehat{S}(\Upsilon) \text{ iff } \Upsilon = (1, 0, 0);$$

$$(P2) \widehat{S}(\Upsilon) \text{ iff } \Upsilon = (0, 0, 1);$$

$$(P3) 0 \leq \widehat{S}(\Upsilon) \leq 1.$$

Proof. Based on the monotonicity of the advanced score function $\widehat{S}(\Upsilon)$, we can obtain that $\widehat{S}(\Upsilon)$ possesses the maximum and minimum values when $\Upsilon = (1, 0, 0)$ and $\Upsilon = (0, 0, 1)$, severally. Consequently, we have $\widehat{S}(\Upsilon)_{\max} = 1$ and $\widehat{S}(\Upsilon)_{\min} = 0$. Accordingly, the proof is achieved.

We shall recompute Examples 1–4 with the novel score function $\widehat{S}(\Upsilon)$ to test its effectiveness, efficiency, and feasibility. The comparison outcomes between the existing works and the novel score function are displayed in Table 1.

In Table 1, we utilize the proposed score function to compute Examples 1–4 and further summarize the computation outcomes acquired by the existing score function S_1, S_2, S_3, S_4 and \widehat{S} . From Example 1, the score functions S_1, S_2, S_3 fail to compare these two SFNs, while the outcome obtained by the proposed score function \widehat{S} is the same as the score function S_4 and H_1 , which shows the reasonability of the presented score function. From Example 2, the score functions S_2, S_3, S_4 fail to compare these two SFNs, while the outcome via the score function \widehat{S} is the same as the score function S_1 , which shows the rationality of the score function \widehat{S} . From Example 3, we can note that

the score function S_1, S_2, S_3 fails to compare SFNs Υ_1 and Υ_2 , and the score function S_4 obtains the counterintuitive outcome. Further, the order of Υ_1 and Υ_2 determined by H_2 is $\Upsilon_1 \succ \Upsilon_2$, which is the same as using the accuracy function H_1 and \widehat{S} , which also proves the superiority of the novel score function. From Example 4, the score function S_2 cannot determine the order of these two SFNs, while the order of SFNs Υ_1 and Υ_2 identified by the novel function \widehat{S} is the same with S_1, S_3, S_4 , which confirms the superiority of the proposed novel function \widehat{S} in distinguishing SFNs.

4 Novel Spherical Fuzzy Aggregation Operators

This section first defines some novel operations for SFNs based on the tangential functions. Then, four improved spherical fuzzy aggregation operators via novel operations are propounded, as well as the associated properties of these operators are studied.

4.1 Novel Operations for SFNs

The aggregation operator is one of the important tools to effectively fuse uncertain information. To achieve the goal of spherical fuzzy aggregation, many operations based on the Archimedean t-norm and t-norms are defined to propose different aggregation operators. Inspired by the work of Rani and Garg [56], this part defined some novel operations of SFNs on the basis of the trigonometric tangent function to enrich the theory of spherical fuzzy integration operators.

Definition 5. Let $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)(t = 1, 2)$ be two SFNs. Then, the novel operations of SFNs based on the trigonometric tangent function are defined as follows:

- (1) $\Upsilon_1 \oplus \Upsilon_2 = (\widetilde{a}(\phi_1, \phi_2), \widetilde{b}(\varphi_1, \varphi_2), \widetilde{b}(\kappa_1, \kappa_2))$,
- (2) $\Upsilon_1 \otimes \Upsilon_2 = (\widetilde{b}(\phi_1, \phi_2), \widetilde{a}(\varphi_1, \varphi_2), \widetilde{a}(\kappa_1, \kappa_2))$,

where $\widetilde{a}(x, y)$ and $\widetilde{b}(x, y)$ are the tangent generation function in reference [56].

The novel operational laws of SFNs using the tangent function are defined as follows.

Definition 6. Let $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)(t = 1, 2)$ be two SFNs. The following operational laws of two SFNs are defined as:

$$\Upsilon_1 \oplus \Upsilon_2 = \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^2 \tan \left(\frac{\pi}{2} (\phi_t)^2 \right) \right)}, \right. \\ \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^2 \tan \left(\frac{\pi}{2} (1 - (\varphi_t)^2) \right) \right)}, \right. \\ \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^2 \tan \left(\frac{\pi}{2} (1 - (\kappa_t)^2) \right) \right)} \right); \tag{3}$$

$$\Upsilon_1 \otimes \Upsilon_2 = \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^2 \tan \left(\frac{\pi}{2} (1 - (\phi_t)^2) \right) \right)}, \right. \\ \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^2 \tan \left(\frac{\pi}{2} (\varphi_t)^2 \right) \right)}, \right. \\ \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^2 \tan \left(\frac{\pi}{2} (\kappa_t)^2 \right) \right)} \right). \tag{4}$$

Theorem 3. Suppose that $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)(t = 1, 2)$, $\Upsilon_1 \oplus \Upsilon_2$ and $\Upsilon_1 \otimes \Upsilon_2$ are also SFN.

Proof. Based on the definition of SFS, we can easily prove that the results of $\Upsilon_1 \oplus \Upsilon_2$ and $\Upsilon_1 \otimes \Upsilon_2$ are SFN.

Theorem 3. For three SFNs $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)(t = 1, 2, 3)$. Then

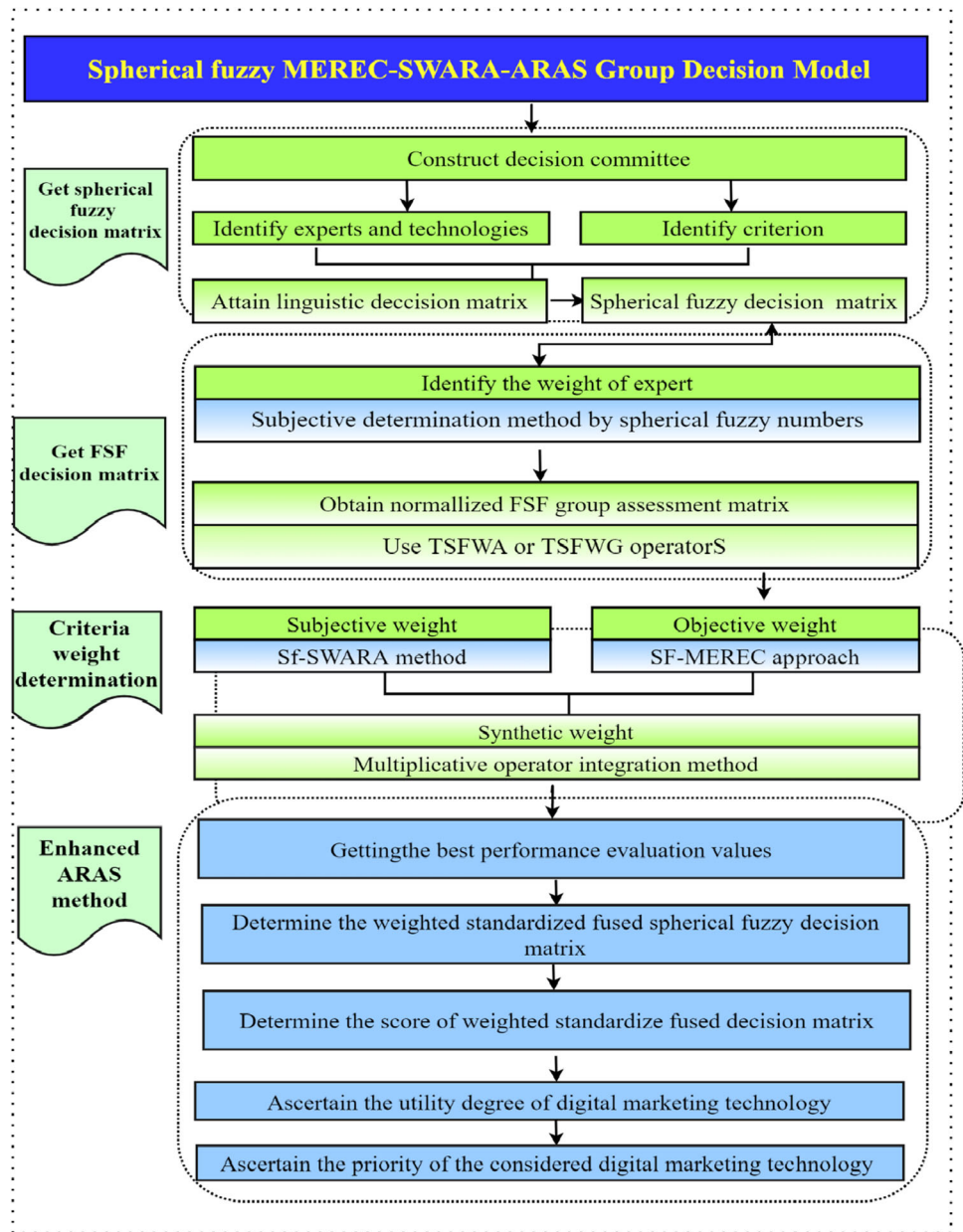
- (1) $\Upsilon_1 \oplus \Upsilon_2 = \Upsilon_2 \oplus \Upsilon_1$;
- (2) $\Upsilon_1 \otimes \Upsilon_2 = \Upsilon_2 \otimes \Upsilon_1$;
- (3) $\Upsilon_1 \oplus (\Upsilon_2 \oplus \Upsilon_3) = (\Upsilon_1 \oplus \Upsilon_2) \oplus \Upsilon_3$;
- (4) $\Upsilon_1 \otimes (\Upsilon_2 \otimes \Upsilon_3) = (\Upsilon_1 \otimes \Upsilon_2) \otimes \Upsilon_3$.

Proof. It is trivial.

In light of the addition and multiplication operations of SFNs in Definition 6, we can obtain the following theorems.

Theorem 4. For an SFN $\Upsilon = (\phi, \varphi, \kappa)$ and a real number α , one has

Fig. 1 The presented SF-MEREC–SWARA–ARAS methodology



$$\begin{aligned}
 \alpha Y_1 &= \underbrace{Y_1 \oplus Y_1 \oplus \dots \oplus Y_1}_{\alpha \text{ times}} \\
 &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (\phi)^2 \right) \right)}, \right. \\
 &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (1 - (\phi)^2) \right) \right)}, \right. \\
 &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (1 - (\kappa)^2) \right) \right)} \right). \tag{5}
 \end{aligned}$$

Proof. It can be proved by mathematical induction easily, it is omitted here.

Theorem 5. For an SFN $Y = (\phi, \varphi, \kappa)$ and a real number α , one has

$$\begin{aligned}
 (\Upsilon_1)^\alpha &= \underbrace{\Upsilon_1 \otimes \Upsilon_1 \otimes \dots \otimes \Upsilon_1}_{\alpha \text{ times}} \\
 &= \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (1 - (\phi)^2) \right) \right)}, \right. \\
 &\quad \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (\phi)^2 \right) \right)}, \right. \\
 &\quad \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (\kappa)^2 \right) \right)} \right). \tag{6}
 \end{aligned}$$

Proof. The proof is similar to Theorem 4.

In view of Theorem 4 and 5, we can derive the scalar multiplication and power operations of SFN in the following.

Definition 7. For an SFN $\Upsilon = (\phi, \varphi, \kappa)$ and a real number α , we have

$$\begin{aligned}
 \alpha \Upsilon_1 &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (\phi)^2 \right) \right)}, \right. \\
 &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (1 - (\varphi)^2) \right) \right)}, \right. \\
 &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (1 - (\kappa)^2) \right) \right)} \right). \tag{7}
 \end{aligned}$$

$$\begin{aligned}
 (\Upsilon_1)^\alpha &= \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (1 - (\phi)^2) \right) \right)}, \right. \\
 &\quad \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (\varphi)^2 \right) \right)}, \right. \\
 &\quad \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\alpha \tan \left(\frac{\pi}{2} (\kappa)^2 \right) \right)} \right). \tag{8}
 \end{aligned}$$

Theorem 6. Ponder two SFNs $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t) (t = 1, 2)$, and real numbers $\alpha, \alpha_1, \alpha_2 > 0$, one has

- (1) $\alpha(\Upsilon_1 \oplus \Upsilon_2) = \alpha \Upsilon_1 \oplus \alpha \Upsilon_2$;
- (2) $(\Upsilon_1 \otimes \Upsilon_2)^\alpha = (\Upsilon_2)^\alpha \otimes (\Upsilon_1)^\alpha$;
- (3) $(\alpha_1 + \alpha_2)\Upsilon_1 = \alpha_1 \Upsilon_1 \oplus \alpha_2 \Upsilon_1$;
- (4) $(\Upsilon_1)^{(\alpha_1 + \alpha_2)} = (\Upsilon_1)^{\alpha_1} \otimes (\Upsilon_1)^{\alpha_2}$.

Proof. It is trivial.

4.2 New Propounded Aggregation Operators

This subsection will put forward several tangent aggregation operators of SFNs, including the TSFWA operator, TSFWG operator, TSFOWA operator, and TSFOWG operator. Let Δ denotes the set of SFNs.

Definition 8. It is assumed that $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t) (t = 1(1)n)$ signifies a family of SFNs and ω_t is the weight of Υ_t such that $\sum_{t=1}^n \omega_t = 1$ with $\omega_t \in [0, 1]$. Then, the TSFWA operator is a mapping $TSFWA : \Delta^n \rightarrow \Delta$, defined as

$$TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) = \omega_1 \Upsilon_1 \oplus \omega_2 \Upsilon_2 \oplus \dots \oplus \omega_n \Upsilon_n. \tag{9}$$

Theorem 7. The fusion outcome of “n” SFNs $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)$ acquired with the TSFWA operator is also an SFN and expressed as

$$\begin{aligned}
 TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\phi_t)^2 \right) \right) \right)}, \right. \\
 &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\varphi_t)^2) \right) \right) \right)}, \right. \\
 &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\kappa_t)^2) \right) \right) \right)} \right).
 \end{aligned}$$

Proof. It is omitted here. since it can be proved by mathematical induction easily.

Several worthwhile properties of the TSFWA operator are discussed below.

Proposition 1 (Idempotency Property) Suppose that Υ_0 be an SFN. If $\Upsilon_t = \Upsilon_0$ for each t . Then

$$TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) = \Upsilon_0.$$

Proof. Since $\Upsilon_t = \Upsilon_0$ for each t , then

$$\begin{aligned}
 &TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) \\
 &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\phi_t)^2 \right) \right) \right)}, \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\varphi_t)^2) \right) \right) \right)}, \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\kappa_t)^2) \right) \right) \right)} \right) \\
 &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\phi_0)^2 \right) \right) \right)}, \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\varphi_0)^2) \right) \right) \right)}, \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\kappa_0)^2) \right) \right) \right)} \right) \\
 &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\tan \left(\frac{\pi}{2} (\phi_0)^2 \right) \right)}, \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\tan \left(\frac{\pi}{2} (1 - (\varphi_0)^2) \right) \right)}, \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\tan \left(\frac{\pi}{2} (1 - (\kappa_0)^2) \right) \right)} \right) \\
 &= (\phi_0, \varphi_0, \kappa_0).
 \end{aligned}$$

Accordingly, $TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) = \Upsilon_0$ holds.

Proposition 2 (Monotonicity Property) Let Υ_t and $\tilde{\Upsilon}_t(t = 1(1)n)$ be two collections of SFNs. If $\phi_t \geq \tilde{\phi}_t, \varphi_t \geq \tilde{\varphi}_t, \kappa_t \geq \tilde{\kappa}_t$ for all t . Then, we have $TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) \geq TSFWA(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n)$.

Proof. As far as

$$TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) = \omega_1 \Upsilon_1 \oplus \omega_2 \Upsilon_2 \oplus \dots \oplus \omega_n \Upsilon_n$$

and

$$\begin{aligned}
 TSFWA(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n) &= \omega_1 \tilde{\Upsilon}_1 \oplus \omega_2 \tilde{\Upsilon}_2 \oplus \dots \\
 &\quad \oplus \omega_n \tilde{\Upsilon}_n.
 \end{aligned}$$

As $\phi_t \geq \tilde{\phi}_t, \varphi_t \geq \tilde{\varphi}_t, \kappa_t \geq \tilde{\kappa}_t$, then $TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) \geq TSFWA(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n)$ always holds.

Proposition 3. (Boundedness Property) Suppose that Υ^+ and Υ^- be the lower and upper bounds of “ n ” SFNs Υ_t . Then, we have

$$\Upsilon^- \leq TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) \leq \Upsilon^+.$$

Proof. Based on the monotonicity and idempotency properties of the TSFWA operator, we have the following:

$$\begin{aligned}
 TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) &\geq TSFWA(\Upsilon^-, \Upsilon^-, \dots, \Upsilon^-) = \Upsilon^-, \\
 TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) &\leq TSFWA(\Upsilon^+, \Upsilon^+, \dots, \Upsilon^+) = \Upsilon^+.
 \end{aligned}$$

Consequently, $\Upsilon^- \leq TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) \leq \Upsilon^+$ holds.

Proposition 4. (Commutativity Property) Let Υ_t and $\tilde{\Upsilon}_t(t = 1(1)n)$ be two sets of SFNs. If $(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n)$ is any permutation of $(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)$. Then

$$TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) = TSFWA(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n).$$

Proof. As we all know that

$$TSFWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) = \omega_1 \Upsilon_1 \oplus \omega_2 \Upsilon_2 \oplus \dots \oplus \omega_n \Upsilon_n$$

$$\begin{aligned}
 TSFWA(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n) &= \omega_1 \tilde{\Upsilon}_1 \oplus \\
 &\quad \omega_2 \tilde{\Upsilon}_2 \oplus \dots \oplus \omega_n \tilde{\Upsilon}_n.
 \end{aligned}$$

Since $(\tilde{\Upsilon}_1, \tilde{\Upsilon}_2, \dots, \tilde{\Upsilon}_n)$ is any permutation of $(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)$, hence the commutativity property for the TSFWA operator is always true.

Definition 10. It is assumed that $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)(t = 1(1)n)$ signifies a family of SFNs and ω_t is the weight of Υ_t , such that $\sum_{t=1}^n \omega_t = 1$ and $\omega_t \in [0, 1]$. Then, the tangent spherical fuzzy ordered weighted averaging (TSFOWA) operator is a mapping $TSFOWA : \Delta^n \rightarrow \Delta$, defined as

$$\begin{aligned}
 TSFOWA(\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n) &= \omega_1 \Upsilon_{o(1)} \oplus \omega_2 \Upsilon_{o(2)} \oplus \dots \\
 &\quad \oplus \omega_n \Upsilon_{o(n)},
 \end{aligned} \tag{10}$$

where $(o(1), o(2), \dots, o(n))$ is an arrangement of $(1, 2, \dots, n)$, such that $\Upsilon_{o(j-1)} \geq \Upsilon_{o(j)}$ for every $j = 2, 3, \dots, n$.

Theorem 8. The fusion outcome of “ n ” SFNs $\Upsilon_t = (\phi_t, \varphi_t, \kappa_t)$ acquired via employing the TSFOWA operator is also an SFN represented as

$$\begin{aligned}
 &TSFOWA(Y_1, Y_2, \dots, Y_n) \\
 &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\phi_{o(t)})^2 \right) \right) \right)} \right), \\
 &\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\varphi_{o(t)})^2 \right) \right) \right)}, \\
 &\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\kappa_{o(t)})^2 \right) \right) \right)}.
 \end{aligned}$$

Proof. Similar to Theorem 6.

The TSFOWA operator also possesses similar propositions to the TSFWA operator, so its introductions and proofs are omitted here. In the following, the weighted geometric operators based on the novel operations are introduced.

Definition 12. It is assumed that $Y_t = (\phi_t, \varphi_t, \kappa_t) (t = 1(1)n)$ signifies a family of SFNs and ω_t is the weight of Y_t , such that $\sum_{t=1}^n \omega_t = 1$ and $\omega_t \in [0, 1]$. Then, the TSFWG operator is a mapping $TSFWG : \Delta^n \rightarrow \Delta$, defined as

$$TSFWG(Y_1, Y_2, \dots, Y_n) = (Y_1)^{\omega_1} \otimes (Y_2)^{\omega_2} \otimes \dots \otimes (Y_n)^{\omega_n}. \tag{11}$$

Theorem 9. The fusion outcome of “n” SFNs $Y_t = (\phi_t, \varphi_t, \kappa_t)$ acquired with the aid of employing the TSFWG operator is also an SFN expressed as

$$\begin{aligned}
 &TSFWG(Y_1, Y_2, \dots, Y_n) \\
 &= \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\phi_t)^2 \right) \right) \right)} \right), \\
 &\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\varphi_t)^2 \right) \right) \right)}, \\
 &\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\kappa_t)^2 \right) \right) \right)}.
 \end{aligned}$$

Definition 13. It is assumed that $Y_t = (\phi_t, \varphi_t, \kappa_t) (t = 1(1)n)$ signifies a family of SFNs and ω_t is the weight of Y_t , such that $\sum_{t=1}^n \omega_t = 1$ and $\omega_t \in [0, 1]$. Then, the tangent spherical fuzzy ordered weighted geometric (TSFOWG) operator is a mapping $TSFOWG : \Delta^n \rightarrow \Delta$, defined as

$$\begin{aligned}
 TSFOWG(Y_1, Y_2, \dots, Y_n) &= (Y_{o(1)})^{\omega_1} \otimes (Y_{o(2)})^{\omega_2} \otimes \dots \\
 &\otimes (Y_{o(n)})^{\omega_n},
 \end{aligned} \tag{12}$$

where $(o(1), o(2), \dots, o(n))$ is an arrangement of $(1, 2, \dots, n)$, such that $Y_{o(j-1)} \geq Y_{o(j)}$ for every $j = 2, 3, \dots, n$.

Theorem 10. The fusion outcome of “n” SFNs $Y_t = (\phi_t, \varphi_t, \kappa_t)$ acquired via employing the TSFOWG operator is also an SFN represented as

$$\begin{aligned}
 &TSFOWG(Y_1, Y_2, \dots, Y_n) \\
 &= \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (1 - (\phi_{o(t)})^2 \right) \right) \right)} \right), \\
 &\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\varphi_{o(t)})^2 \right) \right) \right)}, \\
 &\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\omega_t \tan \left(\frac{\pi}{2} (\kappa_{o(t)})^2 \right) \right) \right)}.
 \end{aligned}$$

Proof. The proof is similar to Theorem 7.

The following properties are easy to obtain by utilizing TSFWG and TSFOWG operators.

Proposition 5. (Idempotency Property) Suppose that Y_0 be an SFN. If $Y_t = Y_0$ for each t . Then

$$\begin{aligned}
 TSFWG(Y_1, Y_2, \dots, Y_n) &= Y_0, \\
 TSFOWG(Y_1, Y_2, \dots, Y_n) &= Y_0.
 \end{aligned}$$

Proposition 6. (Monotonicity Property) Let Y_t and $\tilde{Y}_t (t = 1(1)n)$ be two collections of SFNs. If $Y_t \subseteq \tilde{Y}_t$ for all t . Then, we have

$$\begin{aligned}
 TSFWG(Y_1, Y_2, \dots, Y_n) &\leq TSFWG(\tilde{Y}_1, \tilde{Y}_2, \dots, \tilde{Y}_n), \\
 TSFOWG(Y_1, Y_2, \dots, Y_n) &\leq TSFOWG(\tilde{Y}_1, \tilde{Y}_2, \dots, \tilde{Y}_n).
 \end{aligned}$$

Proposition 7. (Boundedness Property) Suppose that Y^+ and Y^- be the lower and upper bounds of “n” SFNs Y_t . Then, we have

$$\begin{aligned}
 Y^- &\leq TSFWG(Y_1, Y_2, \dots, Y_n) \leq Y^+, \\
 Y^- &\leq TSFOWG(Y_1, Y_2, \dots, Y_n) \leq Y^+.
 \end{aligned}$$

5 The Propounded Methodology

This part designs an innovative MCGDM framework named SF-MEREC–SWARA–ARAS for the analysis of decision problems under spherical fuzzy environments. To be specific, SFNs are utilized to express the cognition preference of experts for the given objectives with respect to the considered criteria. Then, the presented TSFWA operator (or TSFWG operator) is employed to fuse spherical fuzzy preferences given by the experts. Afterward, based on the proposed score function, a comprehensive weighted identification model via integrating SF-MEREC and SF-SWARA methods is developed to estimate the importance of criteria. The combined criteria weight thinks over the removing effect of criteria by the MEREC method and the comparative significance of criteria. As a result, it can acquire more rational weight information and further makes the decision results more credible. In the scheme ranking stage, the ARAS method that utilizes the utility degree theory to determine the prioritization of

$$\tilde{\Phi}^{(l)} = \left(\tilde{\chi}_{st}^{(l)} \right)_{m \times n} = \begin{pmatrix} \left(\tilde{\phi}_{11}^{(l)}, \tilde{\varphi}_{11}^{(l)}, \tilde{\kappa}_{11}^{(l)} \right) & \left(\tilde{\phi}_{12}^{(l)}, \tilde{\varphi}_{12}^{(l)}, \tilde{\kappa}_{12}^{(l)} \right) & \cdots & \left(\tilde{\phi}_{1n}^{(l)}, \tilde{\varphi}_{1n}^{(l)}, \tilde{\kappa}_{1n}^{(l)} \right) \\ \left(\tilde{\phi}_{21}^{(l)}, \tilde{\varphi}_{21}^{(l)}, \tilde{\kappa}_{21}^{(l)} \right) & \left(\tilde{\phi}_{22}^{(l)}, \tilde{\varphi}_{22}^{(l)}, \tilde{\kappa}_{22}^{(l)} \right) & \cdots & \left(\tilde{\phi}_{2n}^{(l)}, \tilde{\varphi}_{2n}^{(l)}, \tilde{\kappa}_{2n}^{(l)} \right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{\phi}_{m1}^{(l)}, \tilde{\varphi}_{m1}^{(l)}, \tilde{\kappa}_{m1}^{(l)} \right) & \left(\tilde{\phi}_{m2}^{(l)}, \tilde{\varphi}_{m2}^{(l)}, \tilde{\kappa}_{m2}^{(l)} \right) & \cdots & \left(\tilde{\phi}_{mn}^{(l)}, \tilde{\varphi}_{mn}^{(l)}, \tilde{\kappa}_{mn}^{(l)} \right) \end{pmatrix},$$

$$l = 1(1)L,$$

wherein $\tilde{\chi}_{st}^{(l)}$ stands for the assessment preference of the scheme $D_s (s = 1(1)m)$ with regard to the criterion $G_t (t = 1(1)n)$ given by the l th expert.

Step 2: Determine the Weight of Experts

The weights of experts are key attention in the construction of MCGDM methodology. It is assumed that the weights of experts are expressed by the linguistic variables and then attain the corresponding SFN. Let $Y_l = (\phi_l, \varphi_l, \kappa_l)$ be an SFN, and then, the weight of the expert f_l can be computed as

$$\gamma_l = \frac{1}{2} \left(\frac{\left(1 + (\phi_l)^2 - (\kappa_l)^2 - ((\varphi_l)^2 + (\delta_l)^2) \right) \times \left(\log_2 \left(1 + (\varphi_l)^2 + (\delta_l)^2 \right) / 100 \right)}{\sum_{l=1}^L \left(1 + (\phi_l)^2 - (\kappa_l)^2 - ((\varphi_l)^2 + (\delta_l)^2) \right) \times \left(\log_2 \left(1 + (\varphi_l)^2 + (\delta_l)^2 \right) / 100 \right)} + \left(\frac{L - \rho_l + 1}{\sum_{l=1}^L (L - \rho_l + 1)} \right) \right), \quad (13)$$

objectives is presented based on the propounded operators and score function. The suggested SF-MEREC–SWARA–ARAS methodology is depicted in Fig. 1.

The detailed decision process of SF-MEREC–SWARA–ARAS is as follows:

Step 1: Establishment the Initial Assessment Matrix

It is supposed that the classical MCGDM problem consists of m scheme denoted as $D = \{D_1, D_2, \dots, D_m\}$ and n assessment criteria indicated as $G = \{G_1, G_2, \dots, G_n\}$. An assessment committee of experts and scholars denoted as $F = \{f_l | l = 1, 2, \dots, L\}$ is invited to provided their opinion for every scheme D_s regarding the criteria G_t via the linguistic variables. Then, the spherical fuzzy decision matrices provided by the experts are obtained by the aid of transforming the linguistic assessments to SFNs denoted as $\tilde{\Phi}^{(l)} = \left(\tilde{\chi}_{st}^{(l)} \right)_{m \times n}$

where γ_l be the weight of the l th expert with the condition $\gamma_l \geq 0, \sum_{l=1}^L \gamma_l = 1$. L stands for the number of experts and $\rho_l (l = 1, 2, \dots, L)$ represents the sorting of every expert attained by the score.

Step 3: Fuse the Expert Decision Matrices

By the determined weight information of the experts, the individual opinion of each expert f_l shall be aggregated to form the fused spherical fuzzy decision matrix (FSF-DM) $\tilde{\Phi} = \left(\tilde{\chi}_{st} \right)_{m \times n}$, which can be determined by utilizing the TSFWA or TSFWAG operators, wherein

$$\begin{aligned} \tilde{\chi}_{st} &= \left(\tilde{\phi}_{st}, \tilde{\varphi}_{st}, \tilde{\kappa}_{st} \right) = TSFWA \left(\tilde{\chi}_{st}^1, \tilde{\chi}_{st}^2, \dots, \tilde{\chi}_{st}^L \right) \\ &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^L \left(\gamma_l \tan \left(\frac{\pi}{2} \left(\tilde{\phi}_{st}^l \right)^2 \right) \right) \right)}, \right. \\ &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\gamma_l \tan \left(\frac{\pi}{2} \left(1 - \left(\tilde{\varphi}_{st}^l \right)^2 \right) \right) \right) \right)}, \right. \\ &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\gamma_l \tan \left(\frac{\pi}{2} \left(1 - \left(\tilde{\kappa}_{st}^l \right)^2 \right) \right) \right) \right)} \right). \end{aligned} \quad (14a)$$

or

$$\begin{aligned} \tilde{\chi}_{st} &= (\tilde{\phi}_{st}, \tilde{\varphi}_{st}, \tilde{\kappa}_{st}) = TSWFG(\chi_{st}^1, \chi_{st}^2, \dots, \chi_{st}^L) \\ &= \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^n \left(\gamma_t \tan \left(\frac{\pi}{2} \left(1 - (\tilde{\phi}_{st}^t)^2 \right) \right) \right) \right)} \right), \\ &\quad \sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^L \left(\gamma_t \tan \left(\frac{\pi}{2} (\tilde{\varphi}_{st}^t)^2 \right) \right) \right)}, \\ &\quad \sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{t=1}^L \left(\gamma_t \tan \left(\frac{\pi}{2} (\tilde{\kappa}_{st}^t)^2 \right) \right) \right)}. \end{aligned} \tag{14b}$$

Step 4: A Combination Weight-Determination Model for Ascertaining Criteria Weight

The weight of assessment criteria is identified by a comprehensive weight-determination model. Let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of criteria with $\omega_t \in [0, 1], \sum_{t=1}^n \omega_t = 1$. To ascertain the weight of the criteria, the improved SF-MEREC model utilizing the novel score function and the SF-SWARA approach is employed for the identification of weight from the subjective and objective perspective. Concretely, the determination process of the model can be expounded as follows:

Case-1: Determinate the objective weight of criteria by improved SF-MEREC model.

The concrete step of the SF-MEREC model using the score function is demonstrated as follows.

Step 4.1: Obtain the normalized FSF-DM.

This step utilizes the simple linear normalization method to scale the FSF-DM $\tilde{\Phi} = (\tilde{\chi}_{st})_{m \times n}$ and derive the normalized FSF-DM denoted as $\Phi = (\chi_{st})_{m \times n}$. It is supposed that G_b and G_c denote the benefit and cost criteria, respectively, and then, the normalized assessment value can be obtained by the following equation:

$$\begin{aligned} \chi_{st} &= (\phi_{st}, \varphi_{st}, \kappa_{st}) = \begin{cases} (\tilde{\phi}_{st}, \tilde{\varphi}_{st}, \tilde{\kappa}_{st}), & t \in G_b \\ (\tilde{\kappa}_{st}, \tilde{\varphi}_{st}, \tilde{\phi}_{st}), & t \in G_c \end{cases}, s \\ &= 1(1)m, t = 1(1)n. \end{aligned} \tag{15}$$

Step 4.2: Figure out the score values.

By means of the propounded spherical fuzzy score function, the score matrix $\Omega = (\sigma_{st})_{m \times n}$ of the normalized assessment value χ_{st} can be computed by

$$\sigma_{st} = \frac{1}{2} \left(1 + (\phi_{st})^2 - (\kappa_{st})^2 - ((\varphi_{st})^2 + (\delta_{st})^2) \times \frac{\log_2(1 + (\varphi_{st})^2 + (\delta_{st})^2)}{100} \right). \tag{16}$$

Step 4.3: Compute the ensemble performance of the schemes.

By the score matrix of the normalized assessments, the overall performance of the schemes D_s can be calculated

$$H_i = \ln \left(1 + \left(\frac{1}{n} \sum_j |\sigma_{st}| \right) \right), s = 1(1)m. \tag{17}$$

Step 4.4: Attain the comprehensive assessment of the schemes by removing criterion.

The assessment of the schemes can be worked out with the help of removing each criterion using the following formulation:

$$H'_{st} = \ln \left(1 + \left(\frac{1}{n} \sum_{k, k \neq t} |\sigma_{sk}| \right) \right), s = 1(1)m, t = 1(1)n. \tag{18}$$

Step 4.5: Estimate the total absolute deviation.

In view of the performance value of schemes determined, the removal effect of the t^{th} criterion denoted as J_t can be calculated by

$$J_t = \sum_s |H'_{st} - H_s|. \tag{19}$$

Step 4.6: Deduce the final criteria weight.

Table 1 Comparison with extant score functions

Score function	Example 1	Example 2	Example 3	Example 4
S_1 [14, 21]	$Y_1 = Y_2$	$Y_1 \succ Y_2$	$Y_1 = Y_2$	$Y_1 \prec Y_2$
H_1 [14, 21]	$Y_1 \prec Y_2$	$Y_1 \succ Y_2$	$Y_1 \succ Y_2$	$Y_1 \succ Y_2$
S_2 [22]	$Y_1 = Y_2$	$Y_1 = Y_2$	$Y_1 = Y_2$	$Y_1 = Y_2$
S_3 [25]	$Y_1 = Y_2$	$Y_1 = Y_2$	$Y_1 = Y_2$	$Y_1 \prec Y_2$
S_4 [24]	$Y_1 \prec Y_2$	$Y_1 = Y_2$	$Y_1 \prec Y_2$	$Y_1 \prec Y_2$
Proposed score function \hat{S}	$Y_1 \prec Y_2$	$Y_1 \succ Y_2$	$Y_1 \succ Y_2$	$Y_1 \prec Y_2$

Table 2 Linguistic variables and their corresponding spherical fuzzy elements [32]

Linguistic variable	SFNs
Absolute high significance (AHS)	(0.9, 0.1, 0.1)
Very high significance (VHS)	(0.8, 0.2, 0.2)
High significance (HS)	(0.7, 0.3, 0.3)
Slightly high significance (SHS)	(0.6, 0.4, 0.4)
Equally significance (ES)	(0.5, 0.5, 0.5)
Slightly low significance (SLS)	(0.4, 0.6, 0.4)
Low significance (LS)	(0.3, 0.7, 0.3)
Very low significance (VLS)	(0.2, 0.8, 0.2)
Absolutely low significance (ALS)	(0.1, 0.9, 0.1)

Based on the absolute deviation, the final criteria weight can be computed by the following formulation as

$$\omega_t^{obj} = \frac{J_t}{\sum_{t=1}^n J_t}. \tag{20}$$

Case-1: Determine the subjective weight by the FF-SWARA method.

Step 4.7 We first obtain the subjective preferences of criteria via each expert using the linguistic variables. Then, the subjective preferences are integrated by the proposed TSFWA operator to obtain the score grades of the final integration value.

Step 4.8 The criteria are sorted via the score grades of the final integration values from high to low.

Step 4.9 Compute the comparative coefficient β_t

$$\beta_t = \begin{cases} 1, & t = 1 \\ q_t + 1, & t > 1, \end{cases} \tag{21}$$

wherein q_t signifies the significance grade of the t th criteria.

Step 4.10 The subjective weight of criteria is calculated by

$$e_t = \begin{cases} 1, & t = 1 \\ \frac{e_{t-1}}{\beta_t}, & t > 1. \end{cases} \tag{22}$$

Step 4.11 The final normalized weight is figured out by

$$\omega_t^{sub} = \frac{e_t}{\sum_{t=1}^n e_t}. \tag{23}$$

Case-3: Identifying the aggregated criteria weights by the SF-MEREC-SWARA method.

The generated weights can capture the objective decision assessment and subjective opinion of the experts. Hence, through the outcomes of objective and subjective weights of the criteria, the ultimate weight is deduced via the non-linear weight integration approach shown in Eq. (24)

$$\omega_t = \frac{\sqrt{\omega_t^{obj} \omega_t^{sub}}}{\sum_{t=1}^n \sqrt{\omega_t^{obj} \omega_t^{sub}}}. \tag{24}$$

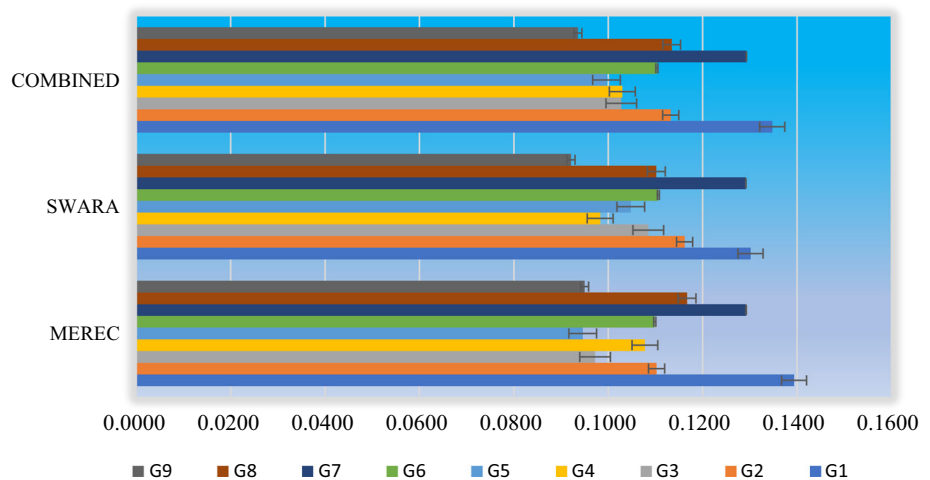
Step 5: Getting the best performance evaluation values.

$$\chi_0 = \begin{cases} \max \chi_{st}, t \in G_b \\ \min \chi_{st}, t \in G_c \end{cases}. \tag{25}$$

Step 6: Discover the weighted standardized FSF-DM.

The weighted standardized FSF-DM $\hat{\mathfrak{R}} = (\hat{\chi}_{st})_{m \times n}$ is ascertained, wherein

Fig. 2 Weights of the criteria using the SF-MEREC-SWARA method



$$\begin{aligned} \widehat{\chi}_{st} &= \left(\widehat{\phi}_{st}, \widehat{\varphi}_{st}, \widehat{\kappa}_{st} \right) = \bigoplus_{t=1}^n \omega_t \chi_{st} \\ &= \left(\sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^L \left(\gamma_l \tan \left(\frac{\pi}{2} (\phi_{st})^2 \right) \right) \right)}, \right. \\ &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^n \left(\gamma_l \tan \left(\frac{\pi}{2} (1 - (\varphi_{st})^2) \right) \right) \right)}, \right. \\ &\quad \left. \sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^n \left(\gamma_l \tan \left(\frac{\pi}{2} (1 - (\kappa_{st})^2) \right) \right) \right)} \right) \end{aligned} \tag{26a}$$

or

$$\begin{aligned} \widehat{\chi}_{st} &= \left(\widehat{\phi}_{st}, \widehat{\varphi}_{st}, \widehat{\kappa}_{st} \right) = \bigotimes_{t=1}^n (\chi_{st})^{\omega_t} \\ &= \left(\sqrt{1 - \frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^n \left(\gamma_l \tan \left(\frac{\pi}{2} (1 - (\phi_{st})^2) \right) \right) \right)}, \right. \\ &\quad \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^L \left(\gamma_l \tan \left(\frac{\pi}{2} (\varphi_{st})^2 \right) \right) \right)}, \right. \\ &\quad \left. \sqrt{\frac{2}{\pi} \tan^{-1} \left(\sum_{l=1}^L \left(\gamma_l \tan \left(\frac{\pi}{2} (\kappa_{st})^2 \right) \right) \right)} \right). \end{aligned} \tag{26b}$$

where $\widehat{\chi}_{st} = \left(\widehat{\phi}_{st}, \widehat{\varphi}_{st}, \widehat{\kappa}_{st} \right)$ be the weighted SFN.

Step 7: Seek out the score of assessment values.

The score index of the weighted standardized FSF-DM can be computed as below

$$\widehat{S}(\widehat{\chi}_{st}) = \frac{1}{2} \left(1 + (\widehat{\phi}_{st})^2 - (\widehat{\kappa}_{st})^2 - \left((\widehat{\varphi}_{st})^2 + (\widehat{\delta}_{st})^2 \right) \times \frac{\log_2 \left(1 + (\widehat{\varphi}_{st})^2 + (\widehat{\delta}_{st})^2 \right)}{100} \right). \tag{27}$$

Step 8: Assess the whole performance value.

The comprehensive performance value of each scheme can be calculated as

$$K_s = \sum_{t=1}^n \widehat{S}(\widehat{\chi}_{st}). \tag{28}$$

Step 9: Seek out the utility grade for each scheme.

The utility grade of each scheme is identified by

$$UG_s = \frac{K_s}{K_0}, \tag{29}$$

Decision results obtain by different weights

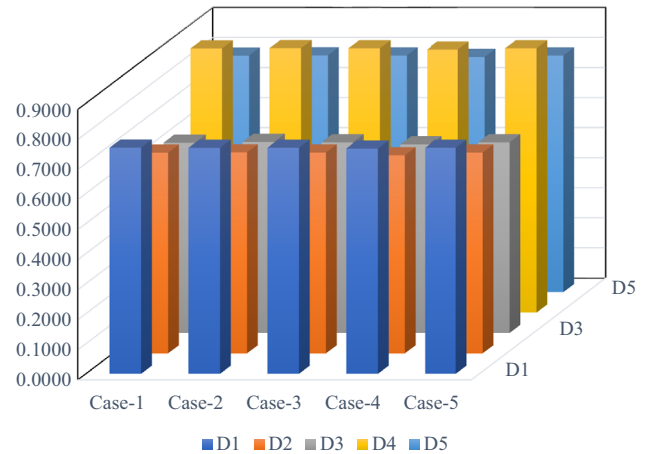


Fig. 3 Decision results obtain by different weights

where $UG_s (0 \leq UG_s \leq 1)$ is the utility grade of the scheme D_s and K_0 is the utility grade of the optimal performance value χ_0 .

Step 10: Sort scheme.

The considered schemes are sorted by the utility grade UG_s , and the most satisfactory option can be achieved by means of

$$K^* = \left\{ D_s \mid \max_{1 \leq s \leq m} UG_s, s = 1, 2, \dots, m \right\}. \tag{30}$$

6 Case Study: Digital Marketing Technology Evaluation

In this section, we will apply the developed SF-SWARA–MEREC–ARAS method to comprehensively evaluate digital marketing technology, as well as further reflect the practicability and effectiveness of the introduced SFS-based decision framework. Then, the sensitivity analysis for the changing of criteria weight is discussed to test the stability of the SF-SWARA–MEREC–ARAS approach.

Fig. 4 Assessment grade of digital marketing technology with different spherical fuzzy methods

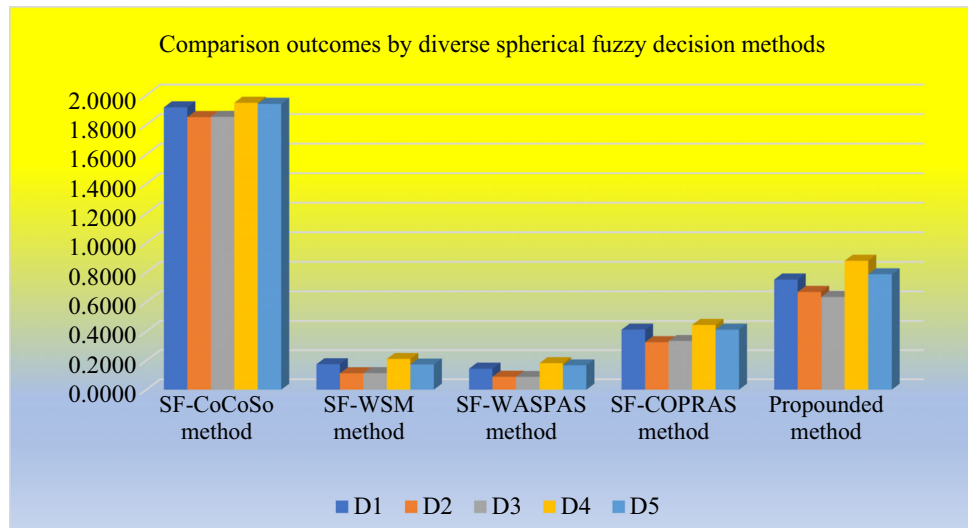


Table 3 The linguistic assessment matrix for digital marketing technology by three experts

	D_1	D_2	D_3	D_4	D_5
G_1	(AHS, HS, SHS)	(HS, AHS, HS)	(VHS, AHS, AHS)	(AHS, VHS, HS)	(HS, VHS, HS)
G_2	(HS, SHS, SHS)	(SHS, HS, ES)	(ES, SHS, SHS)	(VHS, ES, HS)	(HS, VHS, SHS)
G_3	(SHS, ES, HS)	(SLS, ES, SHS)	(ES, SHS, SLS)	(HS, SLS, ES)	(HS, ES, SHS)
G_4	(SHS, SHS, HS)	(ES, SHS, ES)	(HS, SLS, SHS)	(HS, ES, AHS)	(HS, ES, HS)
G_5	(SLS, HS, ES)	(ES, SLS, SHS)	(SHS, ES, SLS)	(SLS, SHS, ES)	(HS, ES, SHS)
G_6	(SHS, SHS, HS)	(ES, SHS, VHS)	(SHS, ES, SHS)	(ES, VHS, ES)	(VHS, SHS, VHS)
G_7	(AHS, VHS, VHS)	(HS, VHS, SHS)	(VHS, SHS, HS)	(SHS, HS, AHS)	(HS, HS, VHS)
G_8	(HS, SHS, SHS)	(VHS, SHS, HS)	(SHS, HS, SHS)	(SHS, VHS, HS)	(HS, HS, VHS)
G_9	(ES, SLS, HS)	(SLS, ES, SLS)	(ES, SHS, ES)	(SHS, SLS, SHS)	(HS, HS, ES)

Table 4 The calculation outcome of expert weights

	f_1	f_2	f_3
Linguistic variable	VHS	HS	SHS
SFNs	(0.8, 0.2, 0.2)	(0.7, 0.3, 0.3)	(0.6, 0.4, 0.4)
Rank	1	2	3
Weight (γ_i)	0.4406	0.3333	0.2261

Meanwhile, the comparative discussion is executed to highlight the strengths of the presented method.

6.1 Decision Analysis for Digital Marketing Technology Evaluation

Digital marketing refers to a new means of product and brand marketing based on Internet user traffic and driven by data intelligence. Relying on digital intelligence technology and digital platform channels, digital marketing helps enterprises to accurately locate and investigate

innovative markets and novel consumers. Its advantage is to communicate with consumers efficiently and economically, thereby greatly improving marketing efficiency. Digital marketing contains the core of technology and is committed to user-deep information mining. Therefore, it is necessary to integrate with network technology to settle and discuss massive data in the marketing process to achieve the precise goal of digital marketing [6]. With the rapid development of network technologies, such as big data, artificial intelligence, and cloud computing, enterprises must select appropriate technologies through different standards of digital marketing models.

This section will carry out the SF-SWARA–MERECA–ARAS method to select the appropriate digital marketing technology for proving the effectiveness and practicability of the SF-SWARA–MERECA–ARAS method. After the preliminary discussion and negotiation of the evaluation team, the evaluation team composed of three experts (f_1, f_2, f_3) identified five technologies for enterprises to select digital marketing including machine learning (D_1),

Table 5 The FSF-DM obtained by the TSFWA operator

	D_1	D_2	D_3	D_4	D_5
G_1	(0.8315, 0.1427, 0.1427)	(0.8163, 0.1567, 0.1567)	(0.8711, 0.1222, 0.1222)	(0.8519, 0.1350, 0.1350)	(0.7417, 0.2521, 0.2521)
G_2	(0.6500, 0.3454, 0.3454)	(0.6223, 0.3690, 0.3690)	(0.5603, 0.4364, 0.4364)	(0.7183, 0.2598, 0.2598)	(0.7274, 0.2615, 0.2615)
G_3	(0.6008, 0.3898, 0.3898)	(0.4892, 0.5033, 0.4267)	(0.5199, 0.4737, 0.4364)	(0.5890, 0.3872, 0.3566)	(0.6264, 0.3623, 0.3623)
G_4	(0.6270, 0.3690, 0.3690)	(0.5375, 0.4592, 0.4592)	(0.6080, 0.3730, 0.3454)	(0.7632, 0.1865, 0.1865)	(0.6506, 0.3387, 0.3387)
G_5	(0.5569, 0.4163, 0.3690)	(0.4986, 0.4949, 0.4364)	(0.5317, 0.4615, 0.4267)	(0.5022, 0.4888, 0.4175)	(0.6264, 0.3623, 0.3623)
G_6	(0.6270, 0.3690, 0.3690)	(0.6401, 0.3254, 0.3254)	(0.5706, 0.4267, 0.4267)	(0.6558, 0.3025, 0.3025)	(0.7572, 0.2311, 0.2311)
G_7	(0.8604, 0.1313, 0.1313)	(0.7274, 0.2615, 0.2615)	(0.7331, 0.2533, 0.2533)	(0.7690, 0.1857, 0.1857)	(0.7294, 0.2650, 0.2650)
G_8	(0.6500, 0.3454, 0.3454)	(0.7331, 0.2533, 0.2533)	(0.6388, 0.3566, 0.3566)	(0.7124, 0.2714, 0.2714)	(0.7294, 0.2650, 0.2650)
G_9	(0.5372, 0.4406, 0.3973)	(0.4370, 0.5610, 0.4267)	(0.5375, 0.4592, 0.4592)	(0.5473, 0.4442, 0.4000)	(0.6679, 0.3247, 0.3247)

big data (D_2), virtual reality technology (D_3), artificial intelligence (D_4), and the Internet of Things (D_5). Due to the relatively small amount of research on digital marketing technology evaluation currently, so we utilize the assessment criteria in reference [4] to conduct the case analysis in this paper. The assessment criteria for assessing

the digital marketing technology are considered from the following three aspects including customer status, company status, and market information, and then, the following nine subcriteria are further pondered to measure the performance of the digital marketing technology, which are explained as customer satisfaction (G_1), customer loyalty (G_2), simultaneous accessibility of product/service (G_3) image (brand value) of company (G_4), promotions (G_5), utilization of social media (G_6), competitive position in market (G_7), market size (G_8), and interaction with competitors (G_9). In the following, SF-SWARA–MEREC–ARAS is employed for selecting the appropriate digital marketing technology.

Based on the professional opinions of the experts, a nine-level language scale from “Absolute high significance” to “absolutely low significance” is provided for experts to express their importance measurement and preference evaluation for the schemes (Table 2). It is supposed that the three linguistic variables denoted are used to signify the weight of experts in the MCGDM process. Further, each technology is evaluated by the three experts over different criteria. The corresponding linguistic

Table 6 The calculation outcome of criteria weight using the spherical fuzzy MEREC method

	D_1	D_2	D_3	D_4	D_5	J_t	ω_t^{obj}
G_1	0.8350	0.8202	0.8716	0.8534	0.7424	0.2838	0.1394
G_2	0.6504	0.6242	0.5603	0.7232	0.7294	0.2245	0.1102
G_3	0.6031	0.5267	0.5382	0.6083	0.6292	0.1980	0.0972
G_4	0.6272	0.5375	0.6236	0.7730	0.6530	0.2195	0.1078
G_5	0.5852	0.5273	0.5487	0.5371	0.6292	0.1927	0.0946
G_6	0.6272	0.6506	0.5703	0.6679	0.7592	0.2237	0.1099
G_7	0.8611	0.7294	0.7356	0.7775	0.7300	0.2632	0.1292
G_8	0.6504	0.7356	0.6392	0.7159	0.7300	0.2377	0.1167
G_9	0.5636	0.5022	0.5375	0.5681	0.6692	0.1934	0.0950

Table 7 The subjective importance of criteria given by expert

Criteria	f_1	f_2	f_3	Fused SFNs	Crisp values
G_1	AHS	VHS	VHS	(0.8604, 0.1313, 0.1313)	0.8611
G_2	VHS	HS	SHS	(0.7397, 0.2491, 0.2491)	0.7416
G_3	HS	SHS	HS	(0.6718, 0.3247, 0.3247)	0.6718
G_4	SHS	ES	SHS	(0.5706, 0.4267, 0.4267)	0.5703
G_5	HS	SHS	ES	(0.6343, 0.3566, 0.3566)	0.6362
G_6	VHS	SLS	ES	(0.6760, 0.2801, 0.2801)	0.6913
G_7	AHS	VHS	HS	(0.8519, 0.1350, 0.1350)	0.8533
G_8	SLS	VHS	HS	(0.6718, 0.2889, 0.2889)	0.6875
G_9	SLS	ES	ES	(0.4598, 0.5381, 0.4480)	0.5034

Table 8 The subjective weight of criteria computed by SF-SWARA method

Criteria	Crisp grade	Comparison significance	Coefficient	Recompute weight	ω_i^{sub}
G_1	0.8611	–	1.0000	1.0000	0.1302
G_7	0.8533	0.0078	1.0078	0.9923	0.1292
G_2	0.7416	0.1117	1.1117	0.8926	0.1162
G_6	0.6913	0.0503	1.0503	0.8498	0.1106
G_8	0.6875	0.0039	1.0039	0.8466	0.1102
G_3	0.6718	0.0157	1.0157	0.8335	0.1085
G_5	0.6362	0.0355	1.0355	0.8049	0.1048
G_4	0.5703	0.0659	1.0659	0.7551	0.0983
G_9	0.5034	0.0669	1.0669	0.7077	0.0921

Table 9 The weighted normalized FSF-DM

	D_0	D_1	D_2	D_3	D_4	D_5
G_1	(0.4559, 0.3313, 0.3313)	(0.3993, 0.3851, 0.3851)	(0.3818, 0.4214, 0.4214)	(0.4559, 0.3313, 0.3313)	(0.4262, 0.3650, 0.3650)	(0.3157, 0.6379, 0.6379)
G_2	(0.2803, 0.6960, 0.6960)	(0.2371, 0.8107, 0.8107)	(0.2238, 0.8331, 0.8331)	(0.1967, 0.8809, 0.8809)	(0.2747, 0.6930, 0.6930)	(0.2803, 0.6960, 0.6960)
G_3	(0.2151, 0.8422, 0.8422)	(0.2040, 0.8635, 0.8635)	(0.1606, 0.9205, 0.8865)	(0.1719, 0.9092, 0.8918)	(0.1990, 0.8617, 0.8372)	(0.2151, 0.8422, 0.8422)
G_4	(0.2911, 0.5574, 0.5574)	(0.2156, 0.8475, 0.8475)	(0.1787, 0.9027, 0.9027)	(0.2072, 0.8507, 0.8266)	(0.2911, 0.5574, 0.5574)	(0.2265, 0.8200, 0.8200)
G_5	(0.2118, 0.8468, 0.8468)	(0.1832, 0.8842, 0.8522)	(0.1615, 0.9200, 0.8950)	(0.1736, 0.9068, 0.8900)	(0.1628, 0.9178, 0.8849)	(0.2118, 0.8468, 0.8468)
G_6	(0.2967, 0.6439, 0.6439)	(0.2231, 0.8372, 0.8372)	(0.2293, 0.7937, 0.7937)	(0.1984, 0.8784, 0.8784)	(0.2369, 0.7652, 0.7652)	(0.2967, 0.6439, 0.6439)
G_7	(0.4301, 0.3625, 0.3625)	(0.4301, 0.3625, 0.3625)	(0.2993, 0.6653, 0.6653)	(0.3032, 0.6501, 0.6501)	(0.3301, 0.5027, 0.5027)	(0.3007, 0.6715, 0.6715)
G_8	(0.2843, 0.6807, 0.6807)	(0.2374, 0.8104, 0.8104)	(0.2843, 0.6807, 0.6807)	(0.2319, 0.8215, 0.8215)	(0.2714, 0.7125, 0.7125)	(0.2819, 0.7016, 0.7016)
G_9	(0.2240, 0.8216, 0.8216)	(0.1703, 0.9033, 0.8801)	(0.1357, 0.9437, 0.8965)	(0.1704, 0.9115, 0.9115)	(0.1740, 0.9050, 0.8817)	(0.2240, 0.8216, 0.8216)

Table 10 The overall utility grade of digital marketing technology

	D_0	D_1	D_2	D_3	D_4	D_5
G_1	0.5465	0.5030	0.4816	0.5465	0.5216	0.3449
G_2	0.2959	0.1990	0.1776	0.1311	0.2964	0.2959
G_3	0.1681	0.1477	0.1197	0.1169	0.1689	0.1681
G_4	0.3849	0.1638	0.1084	0.1794	0.3849	0.1889
G_5	0.1635	0.1533	0.1123	0.1188	0.1215	0.1635
G_6	0.3353	0.1740	0.2107	0.1337	0.2345	0.3353
G_7	0.5242	0.5242	0.3222	0.3333	0.4258	0.3185
G_8	0.3075	0.1993	0.3075	0.1890	0.2819	0.2925
G_9	0.1871	0.1270	0.1071	0.0990	0.1262	0.1871
Overall utility grade	–	2.1912	1.9470	1.8477	2.5618	2.2949
Utility grade	–	0.7522	0.6684	0.6343	0.8794	0.7878
Sorting	–	3	4	5	1	2

Table 11 Decision results obtain by different weights

	Utility grade of digital marketing technologies					Ranking
	D_1	D_2	D_3	D_4	D_5	
Case-1	0.7525	0.6680	0.6332	0.8790	0.7874	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$
Case-2	0.7520	0.6688	0.6354	0.8797	0.7882	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$
Case-3	0.7522	0.6684	0.6343	0.8794	0.7878	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$
Case-4	0.7498	0.6594	0.6274	0.8752	0.7827	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$
Case-5	0.7522	0.6684	0.6343	0.8794	0.7878	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$

Table 12 Decision results obtain by different decision approaches

Methods	Utility grade of digital marketing technologies					Ranking
	D_1	D_2	D_3	D_4	D_5	
SF-CoCoSo approach [57]	1.9180	1.8520	1.8535	1.9490	1.9419	$D_4 \succ D_5 \succ D_1 \succ D_3 \succ D_2$
SF-WSM approach [23]	0.1751	0.1125	0.1137	0.2111	0.1727	$D_4 \succ D_1 \succ D_5 \succ D_3 \succ D_2$
SF-WASPAS approach [23]	0.1443	0.0888	0.0877	0.1818	0.1667	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$
SF-COPRAS approach [58]	0.4124	0.3264	0.3343	0.4443	0.4123	$D_4 \succ D_1 \succ D_5 \succ D_3 \succ D_2$
Propounded method	0.7522	0.6684	0.6343	0.8794	0.7878	$D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$

decision assessments of all technology are shown in Table 3.

By means of Eq. (13), the importance of the three experts can be computed and the results are listed in Table 4. Henceforth, the TSFWA operator or TSFWG operator shown in Eq. (14) is utilized to fuse the individual evaluation opinions for the technologies, and the aggregation assessment information is exhibited in Table 5.

Based upon the FSF-DM in Table 5, the normalized FSF-DM can be obtained by Eq. (15). In this study, the normalization is ellipsised, because all considered criteria are viewed as benefit criteria. The weights of digital marketing technology criteria are determined by the proposed SF-MEREC–SWARA model. First, the score matrix based on the FSF-DM is computed by Eq. (16), and the results are shown in Table 6. Then, the summation of absolute deviations of the criteria is computed based on the overall performances of the digital marketing technology via detaching each criterion using Eqs. (17–18). Hence, the results of absolute deviations of criteria and the final objective weights determined by Eqs. (19–20) are all shown in Table 6.

In addition, the subjective weights of criteria figured out by the SF-SWARA method utilizing Eqs. (21–23) can be acquired and the whole computation outcomes are exhibited in Tables 7–8.

Finally, based on the propounded SF-MEREC–SWARA model, the fused criteria weights can be determined by merging the subjective and objective weights using

Eq. (24). The fused outcomes of the criteria are $\omega_1 = 0.1348$, $\omega_2 = 0.1132$, $\omega_3 = 0.1028$, $\omega_4 = 0.1030$, $\omega_5 = 0.0996$, $\omega_6 = 0.1103$, $\omega_7 = 0.1293$, $\omega_8 = 0.1135$, and $\omega_9 = 0.0936$. Further, the result distributions of the objective weights, subjective weights, and comprehensive weights are visually shown in Fig. 2.

Afterward, based on the score function and FSF-DM, the optimal assessment information for the digital marketing technology under each criterion can be determined by Eq. (25) and given as follows:

$$\chi_0 = [(0.7274, 0.2615, 0.2615), (0.6264, 0.3623, 0.3623), (0.7632, 0.1865, 0.1865), (0.6264, 0.3623, 0.3623), (0.7572, 0.2311, 0.2311), (0.8604, 0.1313, 0.1313), (0.7331, 0.2533, 0.2533), (0.6679, 0.3247, 0.3247)].$$

Next, the weighted normalized FSF-DM can be determined with the aid of Eq. (26), as shown in Table 9. The score value $\widehat{S}(\widehat{\chi}_{st})$ of the weighted normalized assessment values can be computed, based on Table 9 and Eq. (27). The utility grade UG_s of each digital marketing technology D_s can be determined based on the overall assessment value K_s (Table 10). Finally, the prioritization of the digital marketing technologies is determined using Eqs. (28–30) as $D_4 \succ D_5 \succ D_1 \succ D_2 \succ D_3$. The suitable option is the fourth technology, i.e., “artificial intelligence D_4 .”

6.2 Sensitivity Analysis

This subsection analyzes and discusses the sensitivity of the proposed SF-MEREC–SWARA–ARAS methodology. Since the developed approach does not involve parameter issues, we will focus on exploring the fluctuation situation of criteria weights on the ultimate ranking of digital marketing technologies. Next, we will use the following five principles involving subjective weight (Case-1), objective weight (Case-2), combined weight (Case-3), and averaging weight (Case-4), and the weight of all criteria is reduced by 20% (Case-5). Further, the outcomes obtained by the five cases are employed to analyze the robustness of the proposed approach concerning different weights. The calculation results include the utility degree and corresponding ranking of the digital marketing technologies (Table 11 and Fig. 3). Based on the outcomes, we can attain that the ranking results of digital marketing technology obtained by weight information in five different situations are the same, i.e., the optimal option is “artificial intelligence”, while the worst option is “virtual reality”. Accordingly, we can observe that changes in criteria weights have no impact on the ranking of final digital marketing technologies, which can further demonstrate that the method proposed in this article is stable and robust.

6.3 Comparison Study

This subsection will focus on the comparative discussion to further prove the practicalness and rationality of SF-MEREC–SWARA–ARAS in assessing digital marketing technologies. The existing spherical fuzzy decision methods are selected, including the SF-CoCoSo method [57], the SF-WSM approach [23], the SF-WASAS method [23], and the SF-COPRAS method [58]. Then, based on the evaluation information and comprehensive weight of the criteria determined in this article, the above spherical fuzzy decision approaches are applied to solve the evaluation problem of digital marketing technology. The outcomes are given in Table 12 and Fig. 4.

From the comparison results, it is concluded that the rankings of digital marketing technology acquired by different spherical fuzzy decision methods have a bit of difference, while the best option is the fourth technology in all of them. Hence, the effectiveness of the proposed SF-MEREC–SWARA–ARAS decision framework is first proved to develop a reasonable decision. Then, we further compare the inherent differences between the prior and our approaches, which may be the reason for inconsistent sorting results with the proposed method. From the perspective of criterion weight determination, the weight information in the above-mentioned methods is only

ascertained from a subjective or objective perspective or is directly given, which cannot obtain more authentic and comprehensive criterion weight information and thereby reduce the credibility of the decision results. Nevertheless, the SF-MEREC–SWARA method is adopted for the proposed method, which is determined by comprehensively taking into consideration the impact of the decision data itself on the importance of the criterion and the subjective experience of experts. From the perspective of identifying ranking, the SF-WSM approach, SF-WASAS method, and SF-COPRAS method employ algebraic-based operators and prior score functions, which can lead to the irrational ranking comparisons of the schemes. Hence, the proposed method based on novel score function and operators can attain a more accurate utility grade of the schemes with the aid of the enhanced SF-ARAS method.

In addition, the proposed SF-MEREC–SWARA–ARAS method is built up for treating the group decision problems with the incomplete weight of experts, while other methods only address the MCDM problem. Therefore, the developing method is more feasible and generalized than the above approaches in developing decision analysis within an uncertain context.

Considering the aforesaid discussions between the extant spherical fuzzy decision methodologies and the proposed SF-MEREC–SWARA–ARAS method, the merits and strengths of the suggested method for assessing the digital market technology are anatomized as follows:

- The proposed SF-MEREC–SWARA–ARAS method incorporates the SF-MEREC, SF-SWARA, and SF-ARAS within SFSs to cope with the evaluation problems under an uncertain environment, which provides an advanced decision espousal framework for managers to improve the quality of a decision.
- Criteria weights in the proposed model are ascertained through the integration of SF-MEREC and SF-SWARA models, which are presented based on the proposed score function and TSFWA or TSFWG operators. Accordingly, the determination of criteria weight in the developed method can further enhance the accuracy and reliability of the utility ranking outcomes of digital marketing technologies.
- The prioritization of digital marketing technologies is identified by the enhanced SF-ARAS method, which is improved by the novel score function and integration operators. Due to the advantage of the novel score function, the SF-ARAS method can conquer some irrational situations existing in the other methods. The sensitivity and comparison studies also prove the robustness and efficiency of the sorting results attained by our method. Consequently, the introduced SF-MEREC–SWARA–ARAS methodology is more stable,

feasible, and valid than the compared approaches within the SFS context.

7 Conclusion

This study introduced a synthetics assessment framework to evaluate digital marketing technologies and further provided decision support for enterprises and managers. In this regard, a hybrid MCGDM decision methodology was presented with the integration of the SF-MEREC, SF-SWARA, and SF-ARAS models for the prioritization of digital marketing technologies under the SFS environment. In light of this aim, the current study copes with the following attentions associated SFSs. First, the new score function for SFS was proposed to prioritize SFNs and transform them into real numbers, while the related desirable axioms and merits were discussed simultaneously. Second, the TSFWA operator, TSFWG operator, TSFOWA operator, and TSFOWG operator were propounded via the defined new operational laws to aggregate the spherical fuzzy information. Third, the SF-MEREC–SWARA method was propounded to appraise the comprehensive importance of assessment criteria. Fourth, the traditional ARAS method was expanded with the fusion of the score function, TSFWA operator, and SF-MEREC–SWARA with spherical fuzzy settings. Further, this paper carried out a case study about digital marketing technology to confirm the availability and feasibility of the introduced SF-MEREC–SWARA–ARAS decision methodology. The comparison study and sensitivity analyses confirmed that the propounded method was suitable, reliable, and stable within uncertain circumstances.

This study also possesses several limitations in decision analysis. The correlation among the assessment criteria should be thought over in spherical fuzzy MCDM information fusion. Also, the goal weight of experts should be thought about while fusing information. Besides, a reliable degree of assessment information provided by experts should be given for enhancing the reliability of a decision.

Future research might try to conquer the mentioned limitations in our study. Besides, some functional operators, such as Maclaurin symmetric mean [59] and Hamy mean [60], could be proposed to consider the interrelationship among the criteria. The objective weights of experts can also be computed by optimization models and similarity-based methods. The expression of reliability of evaluation information can combine the Z-number theory to represent the assessments comprehensively and the developed method can also be employed in other fields of applications [61–64]. In addition, the consensus-reaching process is also a momentous part of the construction of

group decision methodology which can make the final decision outcomes more reasonable and reliable [65, 66]. Accordingly, the consensus-reaching process, large-group decision-making, and heterogeneous preference information-based problems should be taken into account in the course of group decision analysis under a spherical fuzzy context [67–71].

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Declarations

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