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Optimal Life-Cycle Mortgage and Portfolio Choices in the Presence of the Affordability Constraint

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Abstract

In the past decades, affordability has become a more important issue in housing research. The recent subprime mortgage crisis has shed light on the imperative need to address the affordability challenge. The purpose of this paper is to analyse the housing affordability constraint from a conceptual and analytical standpoint by highlighting its role in a dynamic life-cycle modelling framework. We attempt to assess quantitatively the impact of the affordability constraints on households' optimal consumption, mortgage, portfolio choices and poverty status over the lifetime. Meanwhile, we also investigate the interaction between borrower-based macroprudential policies and social policies aimed at improving poverty and fostering home ownership and credit availability. Based on our simulation results, the mortgage affordability constraints are shown to have a significant impact in the prime and subprime mortgage markets. Moreover, the sensitivity analysis confirms the findings in the baseline case and emphasises that there is a non-linear (hump-shaped) relationship between affordability degrees and housing-caused poverty, particular in the age groups above 40. This implies that in light of the age profile of households and features of mortgage credit markets, the magnitudes of the borrower-based macroprudential policies are needed to be carefully assessed in order to minimise the potential conflicts with other social policies.

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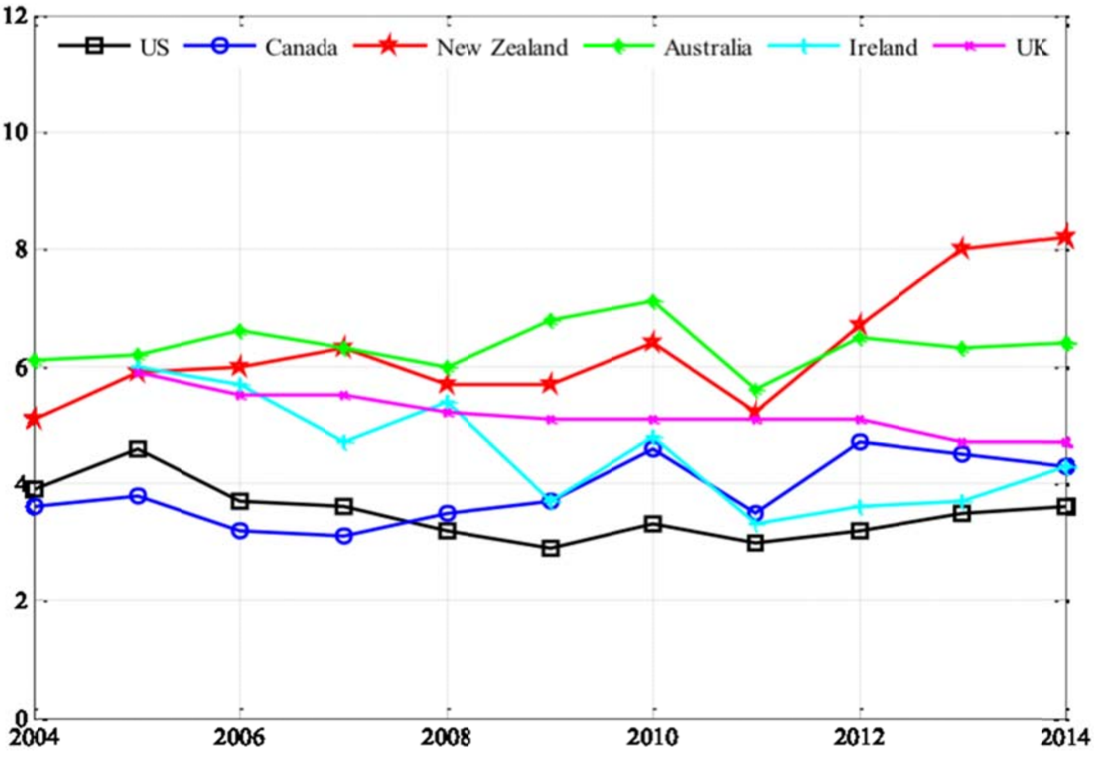
Keywords: Housing affordability, mortgage loan, life-cycle, portfolio choice, housing-caused poverty, macroprudential policy, financial stability

JEL Classification: G01, G21, G11, G18, D10, D91, E27, R29, I38

1. Introduction

In the past decades, the housing affordability issue has received an increasing amount of attention worldwide. In some emerging markets and developed countries, it has become the major topic on the housing policy agenda. The discussions mainly focus on new measurements of affordability, the structure of housing finance, mortgage instrument design and mortgage regulation. The public concern over housing affordability is attributed to two main reasons. First of all, owner-occupied housing is the single largest expenditure item in the budgets of most families. The average household devotes roughly one-quarter of its income to housing expenditure, while poor and young households commonly devote half of their income to housing. These high proportions imply that small percentage changes in housing prices will have large impacts on households' non-housing consumption and asset allocation. Second, the early years of this century were characterised by unprecedented instability in house prices globally (Quigley and Raphael, 2004). There is substantial evidence of a growing housing affordability problem as well as a widening of differences across regions (see Figure 1). This crisis could further induce a severe problem in economic activity and raise concerns about the sustainability of the boom. The recent global financial crisis in 2007–09 triggered by subprime lending has added to these concerns (Gan and Hill, 2009).

Figure 1: Housing Affordability across Countries: 2004–2014



Note: The annual housing affordability in major markets is calculated by the *Demographia International Housing Affordability Survey*. It uses the "Median Multiple" – the median house price divided by the gross annual median household income – to assess housing affordability. This indicator is widely used for evaluating urban markets and is recommended by the World Bank and the United Nations. Housing affordability ratings are assigned as follows: above 5.1: severely unaffordable; 4.1–5.0: seriously unaffordable; 3.1–4.0: moderately unaffordable; below 3.1: affordable. The *11th Demographia International Housing Affordability Survey* provides housing affordability ratings on 86 major markets (with a population of over 1,000,000) and an overall total of 378 markets. This figure shows the average annual Median Multiples in 6 nations in 2004–2014.

Housing affordability issues have been increasingly documented in the research literature. The previous works mainly focus on the definitions and measures of affordability and attempt to examine it in terms of different aspects, such as the deposit gap, borrowing capacity, house-price-to-income ratio and rent-to-income ratio. Bourassa (1996) proposes a borrowing-constraint-based measuring method and applies it to household survey data from 1989–1990 for Australia. He finds that only small percentages of households would have been able to afford homeownership without putting themselves into poverty. Gan and Hill (2009) develop a new approach to the construction of affordability indexes that links to the concept of value-at-risk from the finance literature. As such, the whole distribution of households is taken into account, rather than focusing on either low-income households or the median. Taltavull and Tang (2012) attempts to use a combination of two conventional affordability measures (the rent-to-income ratio and the residual income standards) to examine the affordability problem of social tenants in the English housing association sector. Their analysis confirms the affordability problem in London, where nearly half of the existing housing association tenants fall well below the poverty line.

Despite the fact that a wide range of affordability measures have been developed and applied in different contexts throughout the international arena, "affordability" is still an ambiguous concept as there is no precise and unified definition within the popular press and academic circles. The main reason is that the affordability problem is influenced by a large number of issues in real economic activities. From a more preliminary conceptual point of view, Quigley and Raphael (2004) try to sort out these disparate issues, such as the distribution of housing prices, the distribution of housing quality, the distribution of income, the ability of households to borrow, the public affecting housing markets, the conditions affecting the supply of new or refurbished housing and the choices that people make about how much housing to consume relative to other goods. By doing so, they attempt to sketch policies that might improve the affordability for both homeowners and renters.

However, in response to the recent global financial crisis, it is particularly important to note that the problem of the affordability of mortgages for homeowners accompanied by spiking levels of repossessions, negative equity and bank losses may lead to severe consequences in both mortgage markets and housing markets (Bramley, 2011). Thus, the current affordability measurements centre on the "ability to borrow". In other words, the ability to afford property ownership depends on the household income and the mortgage loan: the higher the household income and/or the lower the mortgage loan, the more affordable the property (Taltavull et al., 2011). Against this background, in the US, major affordability indexes monitored by a variety of organisations are defined as a household's ability to qualify for conventional mortgage financing. For instance, the National Association of Realtors (NAR) index measures the ratio of 25% of the median monthly income to the monthly repayments on a fixed-rate mortgage on the median house at the current interest rates. The US Department of Housing and Urban Development (HUD) index computes the ratio of the median family income to the income required to qualify for a conventional mortgage on the median-valued house sold.[1] In this context, some economists analyse particularly the interrelationships between housing affordability and the mortgage market. For instance, Taltavull et al. (2011) analyse empirically the relationship between mortgage liquidity and housing affordability in Northern Ireland during the boom–bust cycle in the residential property market. They find that the affordability has been driven by the deregulation of the mortgage market, contributing to the rise in house prices and affordability pressures during the market up cycle in Northern Ireland.

[1] The third major index in the US is the National Association of Home Builders (NAHB) index. It measures the fraction of dwellings sold that could be purchased by the median household with 28% of the household income.

Our paper is basically related to two strands of literature. The first strand, concerning affordability, has been introduced in the preceding paragraph. The second relevant strand of literature addresses dynamic life-cycle portfolio choices in the presence of owner-occupied housing. The prior research on owner-occupied housing and portfolio choices began with the work of Grossman and Laroque (1990), who focus on the impact of the transaction cost of illiquid durable goods. Brueckner (1997) and Flavin and Yamashita (2002) use a static, one-period, mean-variance framework to study the relation between housing and investors' optimal holding of financial assets. The studies by Cocco (2005) and Yao and Zhang (2005a) are the first to investigate optimal housing, consumption and portfolio decisions in a dynamic life-cycle context. Other researchers extend their work in various directions. Van Hemert (2006) integrates the bond market and allows for an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM) and a combination of the two (a hybrid mortgage). Hu (2005) and Yao and Zhang (2005b) consider more realistic scenarios in the housing market, including costly mortgage refinancing and default penalties. Recently, Campbell and Cocco (2015) study the mortgage default decisions in the rational life-cycle model. Specifically, they analyse households' decisions for endogenously determined mortgage rates. Incorporating risks to labour income, house prices, inflation, and interest rates, they attempt to understand the types of mortgage that borrowers take out and their subsequent decision to refinance, cash out, or default on those mortgages. Unlike these previous papers, we extend the earlier work of Cocco (2005) by adding the housing affordability constraint explicitly in an attempt to study the relationship between households' affordability and mortgage loan and to explore how the housing affordability constraint affects households' optimal consumption, mortgage, portfolio choices and poverty status at different ages.

Our findings are threefold. Firstly, we confirm the robustness of the main results of Cocco (2005) and Hu (2005). They argue that integrating housing into standard life-cycle models can partially resolve the portfolio composition puzzle.[2] For many younger homeowners, investment in housing keeps their cash-on-hand low. They are relatively poor and need to convert all their spendable resources into housing assets, leaving less/no savings for financial investment. Housing investments crowd out stock investments. As homeowners age, housing equity (the house value net of debt) and human capital are more important relative to stocks and bonds for future consumption, since households on average have less mortgage liability in this period of life. Consequently, older homeowners are more willing to take risks in their liquid financial portfolio (Cocco, 2005). In the presence of the affordability constraint, we show that their findings still hold qualitatively. Quantitatively, our simulation results are more successful in matching the observations in the data.[3]

Secondly, our paper makes a contribution to the affordability literature. According to the simulation results, the housing affordability constraint has a significant impact on optimal consumption, portfolio composition, mortgage choices and poverty status. Massive borrowing easily puts households into poverty, especially those under the age of 40. Under the assumption that the affordability constraint is strictly taken into account, households manage to maintain a fairly smooth consumption pattern and keep the mortgage debt burden spread more evenly throughout their lifetime. Another purpose of this paper is to assess quantitatively the influence of important factors concerning housing affordability in the model. More specifically, we focus primarily on the mortgage market and distinguish two types: "contracted" and "liberalised" (Taltavull et al., 2011). The contracted mortgage market is close to the

[2] Basic (standard) theoretical portfolio choice models without housing predict that the risky asset share is expected to decline with age. Nevertheless, much empirical evidence is at odds with the predictions: risky asset shares among young people are generally found to be low and hump-shaped, increasing or constant with age. Furthermore, young and poor households are usually borrowing-constrained and tend to hold less stock. (See Ameriks and Zeldes, 2004; Campbell, 2006; Guiso et al., 2003; Haliassos and Michaelides, 2002; Heaton and Lucas, 2000). These contradictions are regarded as "the composition puzzles".

[3] For detailed explanations, please see section 3.1.

traditional prime (standard) mortgage market, in which the borrower cost is primarily driven by the down payment alone, given that the minimum credit history requirements are satisfied. By contrast, a liberalised mortgage is similar to a subprime mortgage and it is normally made out to higher-risk borrowers who buy pricey houses relative to their income level and make little or no down payment yet have a high-interest rate. The main benefits of this type of mortgage are the increased numbers of homeowners and the opportunity for these homeowners to create wealth (Chomsisengphet and Pennington-Cross, 2006). Therefore, in our simulation exercises, we use the down-payment ratio and mortgage rate to characterise these mortgages' lending patterns. Instead of deriving the optimal mortgage contract, as in Piskorski and Tchistyi (2010, 2011), we attempt to illustrate quantitatively the impact of the affordability constraint in the subprime mortgage market as well and compare it with that in the prime market.

Thirdly, our paper also makes a contribution to the macroprudential policies literature. In the numerical experiments, we focus specifically on loan-to-value (LTV) and loan-to-income (LTI) ratio. The main reason is that these measures are of particular importance in the sense of macroprudential policies. Broadly speaking, macroprudential policy is seen as aiming at financial stability. In terms of the specific goals of macroprudential policy, the general view is that it is all about limiting the risk and costs of systemic crises. Since boom-bust cycles in real estate markets have been major factors in systemic financial crises and therefore need to be at the forefront of macroprudential policy (Hartmann, 2015). Macroprudential policies are usually used according to whether they are aimed at borrowers (LTV and LTI ratios), banks' assets or liabilities (limits on credit growth, foreign currency credit growth and reserve requirements) and policies that encourage counter-cyclical buffers (counter-cyclical capital, dynamic provisioning and profits distribution restrictions) (Claessens et al., 2013). Recently, a new empirical literature on the effectiveness macroprudential regulatory policies seems to increasingly suggest that borrower-based instruments may be more effective in containing real estate bubbles than bank-based instruments (Hartmann, 2015). While macroprudential policies aim at financial stability, there are clear implications for housing affordability. As house prices and debt levels trend increasingly upwards, residential housing becomes less affordable, particular for first-time buyers. In the short-run, macroprudential policy tools tend to make credit less accessible; however, Spencer (2013) , Deputy Governor and Head of Financial Stability of the Reserve Bank of New Zealand indicated that macroprudential polices should make house prices more affordable in the long-run and reduce the risks of a sharp housing downturn. Against this background, we also attempt to scrutinise the interaction between borrower-based macroprudential policies and social policies aimed at improving affordability/poverty status and fostering home ownership and credit availability.

According to the numerical analysis, the average optimal LTI and LTV ratios in the subprime market decline strongly in the presence of affordability restrictions, keeping consumption, loans and poverty status fairly stable over the lifetime. Moreover, the mechanism of the affordability constraint works more effectively in the subprime market than in the prime market. Finally, our sensitivity analysis confirms the robustness of the main findings in the baseline model. It also emphasises that the magnitudes of the constraint influence households' behaviour in different ways. According to the results, we find a non-linear (hump-shaped) relationship between affordability degrees and poverty status. The probable reason is that a less strict affordability restriction allows people to accumulate more resources on hand, especially after the age of 40. As such, they could use them to smooth the consumption and rebalance their portfolios, which reduce the risk of putting themselves into poverty. The paper proceeds by first presenting the life-cycle modelling set-up in section 2. In section 3, we provide the parameterization of the model and conduct a simulation and sensitivity analysis. Finally, section 4 concludes.

2. Model

In this section, we extend a realistic dynamic life-cycle model to study how the housing affordability constraint affects households' non-housing and housing consumption, portfolio choices in financial markets and poverty status throughout their lifetime. We consider a partial equilibrium model in which the households take all the prices as given.

2.1 Preferences

Putting overlapping generation aspects aside, we only need one index t to denote adult age. We assume that a representative household enters the model at the age of 30 and works for the first K periods and leaves our model after T periods. We allow for uncertainty during the household's life by taking the age-specific conditional survival probabilities π_t into account, where π_t denotes the probability that the household will be alive in period $t+1$, conditional on being alive in the previous period t (Hubbard, Skinner and Zeldes, 1995). In each period, the household needs to choose non-durable goods consumption C_t and housing services H_t. While the former needs to be purchased anew in every period, housing services depend on the size of the house stock that the household chooses to own. More specifically, a household receives one unit of housing service for every unit of owned housing stock. The household is assumed to be concerned about its expected discounted lifetime utility from both housing services and non-durable goods. The preferences are given by:

$$E_1\left[\sum_{t=1}^{T}\beta^{t-1}\left(\prod_{j=0}^{t-1}\pi_j\right)u(C_t,H_t)\right], \text{ where } u(C_t,H_t)=\frac{({C_t}^{1-\theta}{H_t}^{\theta})^{1-\gamma}}{1-\gamma}. \quad (1)$$

β is the time discount factor, γ is the coefficient of relative risk aversion and θ determines the housing service shares in a static model.

2.2 Labour Income

In each working period ($t \leq K$), the household receives an exogenous stochastic stream of labour income Y_t. Let $y_t \equiv lnY_t$, and y_t is commonly modelled as the sum of a permanent component and a transitory shock ε_t. The permanent component consists of a deterministic function $f(t,Z_t)$ of age t and individual characteristics Z_t and of a persistent income component υ_t following an AR(1) process:[4]

$$\upsilon_t = \phi\upsilon_{t-1} + u_t. \quad (2)$$

[4] This assumption follows Hubbard, Skinner and Zeldes (1995). They estimate a general first-order autoregressive process and find the auto-correlation coefficient to be very close to one, indicating that $\phi \approx 1$. For convenience, we will use this value in the numerical analysis.

Therefore:

$$y_t \equiv lnY_t = f(t, Z_t) + \upsilon_t + \varepsilon_t, \ \ \forall\, t \le K, \tag{3}$$

where $\varepsilon_t \sim i.i.dN(0,\ \sigma_\varepsilon^2)$, $u_t \sim i.i.dN(0,\ \sigma_u^2)$ and u_t is uncorrelated with ε_t. Since most of the uncertainty related to future labour income in retirement has been resolved, the income at this stage of life is then modelled simply as a constant fraction of the permanent labour income in the last working period:

$$y_t = \ln(\zeta) + f(K, Z_K) + \upsilon_K, \ \ \forall\, K < t \le T, \tag{4}$$

where ζ is the replacement ratio. We will explain the detailed numerical implementation of stochastic labour income in section 3.1.

2.3 Housing

Owner-occupied housing differs from other financial assets in that housing is both a durable consumption good and an investment instrument. Unlike traditional liquid financial assets, the housing asset enters an investor's wealth accumulation in a complicated way because it involves a down payment, mortgage debt and adjustment and maintenance costs. The choice about how much housing and which house to buy is a joint consumption–investment decision. To purchase a house, a fraction of the market house value is required as a down payment and the rest of the cost can be financed with a mortgage loan. Notice that mortgage payments over a long horizon are needed out of an uncertain stream of labour income, making the post-mortgage-payment income lower and more volatile. Once a house has been bought, there is less flexibility in housing expenditure since the maintenance costs and transaction costs associated with frequent trades can be very large. Therefore, we assume that the maintenance costs and transaction costs are equal to a proportion ψ and λ of the current house value, respectively. The price per unit of housing in period t is denoted by P_t, such that the market value of a house of size H_t is then equal to $P_t H_t$.

In this paper, we don't consider the rental market. In fact, rental market has been discussed vividly previously. Hu (2002) studies portfolio choices for homeowners in the presence of a house rental market in a five-period model. In her setup, investors make portfolio and housing adjustments every 10 years. They are not allowed to own in the first period so that they can accumulate enough wealth. Further, there is no mortality prior to the final period or bequest motive. Yao and Zhang (2005a) extend Grossman and Laroque (1990) by incorporating a nondurable numeraire consumption good, housing price risk, collateral requirements, and an uninsurable stochastic labour income. Besides, they explicitly introduce the house rental market for housing services. It allows investor to separate their housing consumption choice from their housing investment choice and to consume housing services while saving toward the down payment for a house of the desired size. However, in response to the recent global financial crisis, we are more interested in the problem of the affordability of mortgages for homeowners, in particular, households' "ability to borrow mortgage loan", since their choices play a big part in maintaining a high or low housing cost burden. Therefore, we assume that there is a minimum house size H^{min} that $H_t \ge H^{min}, \ \forall\ 1 \le t \le T$ for each household. By doing so, the rental market is eliminated (Cocco, 2005).

For computational simplicity, we model the house price as an exogenous stochastic process with a deterministic exponential trend. Let $p_t = \ln(P_t)$, then the detrended log price of housing per unit is defined as:

$$p'_t \equiv p_t - gt. \tag{5}$$

where g is the real log house price growth rate. Although the house price is assumed to be an exogenous source, we should not neglect its role in the model. First of all, the house price is an important determinant of the affordability of homeownership. Some quantitative measures of housing affordability can be viewed as a relationship between home prices and household incomes. Rising home prices impede prospective households' accumulation of a down payment and raise other required housing expenses. Second, the price of housing in a given region is frequently affected by labour income shocks in the same region (Linneman and Megbolugbe, 1992). Cocco (2005) assumes that cyclical fluctuations in house prices are perfectly correlated with aggregate (permanent) labour income shocks:

$$v_t = \kappa_v p'_t, \tag{6}$$

where κ_v is the regression coefficient. However, in our baseline case, we abstract from this complication to make the model tractable.

2.4 Financial Assets

We assume that there are three financial assets in this model. A riskless asset, called bonds, has a constant gross real return $R_f = (1 + r_f)$. The risky asset, called stock, has a stochastic gross real return R_t^s, and its excess return is given by:

$$R_t^s - R_f = \mu + \eta_t, \tag{7}$$

where μ is the mean equity premium and η_t is the innovation to excess returns in period t and is assumed to be $\eta_t \sim i.i.d N(0,\ \sigma_\eta^2)$. We denote the amount of bonds and stocks that the investor has in period t by B_t and S_t, respectively, and assume that the investor faces the following borrowing and short-sale constraints:[5]

[5] The borrowing constraint (8) ensures that the investor's allocation to bills is non-negative on all the dates. It prevents the investor from capitalizing or borrowing against future labour income or retirement wealth. The short-sale constraint (9) ensures that the investor's allocation to equities is non-negative on all the dates.

$$B_t \geq 0, \tag{8}$$

$$S_t \geq 0. \tag{9}$$

Let α_t represent the proportion of liquid assets invested in stocks over the sum of bonds and stocks in period t. Then, these two constraints imply that $\alpha_t \in [0, 1]$ for $\forall\, t$. The third financial asset is the mortgage loan M_t. To become a homeowner, an investor needs to make a down payment, which is assumed to be a proportion δ of the housing value at loan initiation. To finance the rest, the investor can borrow a mortgage loan against his house with a constant gross real rate $R_M = (1 + r_M)$. The mortgage is assumed to mature at T, such that the entire balance is paid off in the terminal period. Therefore, a newly initiated mortgage satisfies:[6]

$$0 \leq M_t \leq (1 - \delta)P_t H_t, \;\; \forall\, 1 \leq t \leq T. \tag{10}$$

For convenient numerical implementation, we allow for costless renegotiation of the desired level of debt in each period. By doing this, the mortgage will be treated as a control variable but not a state variable in the computation.[7]

2.5 Housing Affordability Constraint

Investors –in other words, home buyers– must have sufficient wealth and income to gain access to mortgage loans. Earlier studies of housing tenure choice in the US (Linneman and Wachter, 1989) and Australia (Bourassa, 1995) show that households' borrowing constraints are significant determinants of their access to homeownership. A useful starting point for understanding housing affordability is the concept of an affordability constraint, which is similar to the "borrowing constraint" and "affordable limit" introduced by Bourassa (1995) and Gan and Hill (2009), respectively. The affordability constraint is defined as the ratio of the allowable loan to the maximum income allocated to the mortgage. More formally, the mortgage loan M_t is deemed affordable for a household with gross income Y_t in each period if it satisfies:

[6] In practice, the loan-to-value ratio constraint $0 \leq M_t \leq (1 - \delta)P_t H_t, \;\; \forall\, 1 \leq t \leq T$ only applies at loan origination. When there is a housing market downturn, for instance, caused by a sharp decline of house prices, the investor can carry a mortgage larger than the market value of his house. In this case, the investor would be better off defaulting on his mortgage obligation theoretically. However, according to Deng, Quigley, and Van Order (2000), residential mortgage defaults are rare and less than 2% in reality. Against this background, we didn't consider the mortgage default. According to our model, a rational utility-maximizing household would rather continue to borrow a very small amount of mortgage loan than defaulting. Therefore, in the Matlab code we optimized the mortgage loan M_t, such that cash-on-hand ≥ non-housing consumption + housing expenditure − mortgage, i.e. cash-on-hand - non-housing consumption - housing expenditure + mortgage must be larger or equal to 0. And accordingly, the value of the mortgage could be zero.

[7] In fact, households could consider the mortgage debt as an additional resource, so that they can use it either for non-housing consumption and participation in the equity market or to invest in the housing market. However, under the assumption that mortgage debt is costlessly renegotiated in each period, an investor will never hold bonds and debt simultaneously.

$$\frac{M_t}{\omega Y_t} \leq \text{Affordability Constraint (AC)}, \quad \forall\, 1 \leq t \leq T, \tag{11}$$

where ω denotes the maximum proportion of gross income required to qualify for a mortgage. Otherwise, the house is deemed unaffordable. Naturally, the consideration of ω is that its value might vary with income. However, ω does not seem to change much empirically with the income level. Piazzesi et al. (2007) find that in the US the lowest income quintile spends roughly the same percentage as the highest income quintile based on the data from the consumer expenditure survey for the years 1984–2002. We thus assume that the household buys a house and the present values of the maximum affordable labour income stream and mortgage loan are:

$$\sum_{n=1}^{T} \frac{\omega E_1[Y_n]}{R_f^{n-1}}, \tag{12}$$

$$\sum_{n=1}^{T} \frac{E_1[M_n]}{R_M^{n-1}}. \tag{13}$$

Accordingly, the affordability constraint (AC) is obtained as follows:

$$\frac{M_t}{\omega Y_t} \leq AC\,, \qquad \forall\, 1 \leq t \leq T. \tag{14}$$

where $AC = \frac{R_f - \left(\frac{1}{R_f}\right)^{T-1}}{R_M - \left(\frac{1}{R_M}\right)^{T-1}} \frac{(R_M - 1)}{(R_f - 1)}$.[8] In other words, the average mortgage–income ratio should not be larger than ωAC in each period in the model.

2.6 Budget Constraints and Dynamic Optimization

In each period t, the household uses its total spendable resource or *cash-on-hand* Q_t, which consists of stochastic labour income Y_t and liquid wealth LW_t, to purchase non-housing goods C_t, to adjust its housing stock H_t or to invest in liquid assets S_t and B_t. It is expected that the household invests in liquid assets after paying for consumption and housing. The period t budget constraint is given by:

$$Q_t = \begin{cases} C_t + B_t + S_t + \psi P_t H_{t-1} - M_t, & No\ Adjust \\ C_t + B_t + S_t + (1 + \psi + \delta) P_t H_t - (1 - \lambda) P_t H_{t-1} - M_t, & Adjust \end{cases} \tag{15}$$

where

[8] The detailed construction process of affordability constraint is given in Appendix A.

$$Q_t = LW_t + Y_t, \tag{16}$$

$$LW_t = B_{t-1}\, R_f + S_{t-1}\, R_t^s - M_{t-1} R_M. \tag{17}$$

The household maximises its expected discounted lifetime utility (1) subject to budget constraints (15)–(17), the borrowing constraint for a mortgage (10), the affordability constraint (14) and no short sales for bonds (8) and stocks (9) on all the dates. A household's state depends on its beginning-of-period liquid wealth, realised labour income, current house price and size of the existing housing, that is:

$$X_t \equiv \{\, LW_t,\ Y_t,\ P_t, H_{t-1}\,\}. \tag{18}$$

Meanwhile, the control variables of this problem are given as:

$$A_t \equiv \{\, C_t,\ H_t,\ M_t,\ B_t,\ S_t\,\}. \tag{19}$$

Accordingly, the Bellman equation of the household's intertemporal consumption and investment problem can be written as:

$$V_t(X_t) = \max_{A_t} \left\{ \frac{\left({C_t}^{1-\theta} {H_t}^{\theta}\right)^{1-\gamma}}{1-\gamma} + \beta E_t[V_{t+1}(X_{t+1})] \right\}, \qquad \forall\, 1 \le t \le T. \tag{20}$$

For simplicity of implementation, it is possible to standardise this maximisation problem such that some state variables vanish. The standardisation of the life-cycle portfolio choice model is most commonly performed by normalising the problem by the permanent component of labour income $Y_t{}'$(Cocco, Gomes and Maenhout, 2005; Gomes and Michaelides, 2003, 2005; Polkovnichenko, 2007).[9] As a result of this standardization, the vectors of the state and control variables become:

$$x_t = \{q_t,\ h_t'\}, \tag{21}$$

$$a_t = \{c_t,\ h_t,\ m_t,\ \alpha_t\}, \tag{22}$$

where $q_t = Q_t / Y_t{}'$ is the wealth–income ratio, $h_t' = P_t H_{t-1} / Y_t{}'$ is the household's beginning-of-period house-value-to-income ratio, $c_t = C_t / Y_t{}'$ is the consumption-to-income ratio, $h_t = P_t H_t / Y_t{}'$ is

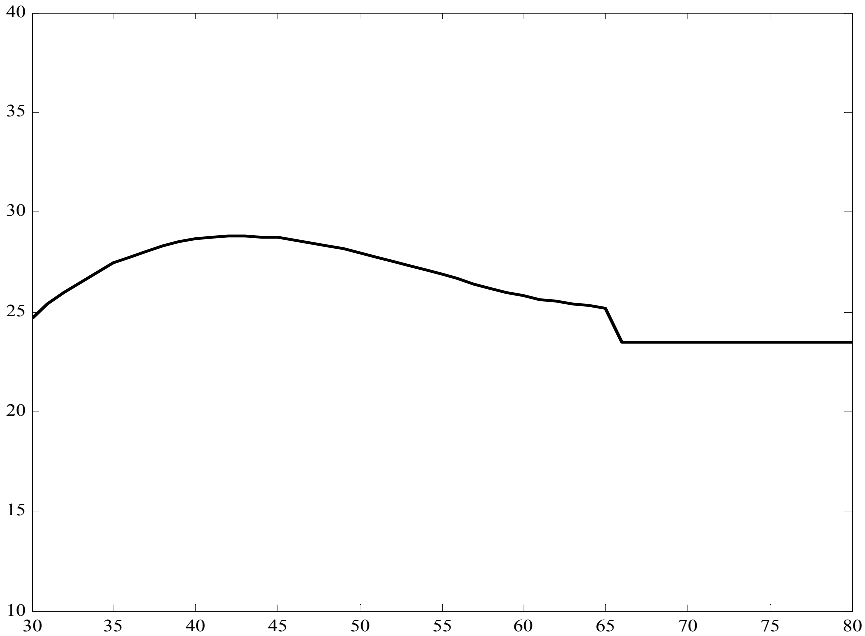
[9] According to the setting up of labour income, $Y_t{}'$=exp(υ_t), where υ_t is the permanent income shock.

the house-value-to-income ratio, $m_t = M_t/Y_t'$ is the mortgage-to-income ratio and $\alpha_t = S_t/(S_t + B_t)$ is the proportion of liquid assets invested in stocks over the sum of bonds and stocks. However, there is no general analytical solution to this optimization problem. We can only solve it numerically using backward induction (Judd, 1998).[10]

3. Numerical Analysis

Before we start the numerical analysis, a detailed parameterization needs to be specified. Fortunately, previous researchers have already estimated the relevant parameters based on data in the US, which are widely used to solve the life-cycle portfolio selection problem in the presence of housing.[11] Therefore, we rely fully on their results instead of estimating our own. In the baseline case, we set the annual discount factor β as 0.96 and the coefficient of relative risk aversion γ as 5. The weight that housing carries in the instantaneous utility function is set as 0.1 (Cocco, 2005), and the conditional survival probabilities π_t are parameterized based on the data from the National Centre for Health Statistics. We assume that the household makes the decision annually starting at the age of 30 and leaves the model at the age of 80.[12]

Figure 2: Simulated Household Income by Age



The household's 50 years of life are divided into two stages: working and retirement. During the first stage (age 30–65), the household's income is determined by a permanent component and a transitory shock. The deterministic function $f(t, Z_t)$ is assumed to be additively separable in t and Z_t. The vector Z_t represents personal characteristics other than age, and the fixed household effect includes marital status and household size. To obtain the profiles for the numerical solution, we follow Cocco,

[10] The detailed standardizing of the optimal problem and the numerical techniques are given in Appendix A.
[11] Campbell and Cocco (2015), Cocco (2005), Cocco, Gomes and Maenhout (2005), Hu (2005) and Yao and Zhang (2005a) estimate the relevant parameters based on US data.
[12] Since we eliminate the rent market and assume that the household holds at least H^{min} housing stock before entering the model, it is reasonable to set the starting age of the household as 30. This assumption is also consistent with Yao and Zhang's (2005a) findings. Before 30, the fraction of households owning a house is almost zero.

Gomes and Maenhout (2005), who fit a third-order polynomial to the age dummies for each education group: no high school degree, high school degree and college degree. The coefficients are estimated using the Panel Study of Income Dynamics (PSID) data. In the baseline model, we use the wage profile of a household with a college degree. After retirement (age 65–80), the household receives a constant annual income determined by the replacement ratio ζ. It is calibrated as the ratio of the average of the labour income variable defined for retirees to the average of the labour income in the last working year prior to retirement. The error structure of the labour income process is obtained by following Carroll and Samwick's (1997) method, and the variance of permanent and transitory labour income shocks are taken from Cocco (2005) and set as 0.019^2 and 0.133^2, respectively. Based on these values, Figure 2 illustrates the average household's income by age for 50,000 simulation trials.[13]

Table 1: Baseline Parameters

Parameter and Description	Value
Time discount parameter (β)	0.96
Curvature parameter (γ)	5.0
Preference for housing (θ)	0.1
Retirement age (K)	65
Maximum age (T)	80
Replace. rate (ζ)	0.9388
Variance of permanent shocks (σ_u^2)	0.019^2
Variance of transitory shocks (σ_ε^2)	0.133^2
Real log house price growth (g)	0.01
Variance of detrended log house price ($\sigma_{p'}^2$)	0.062^2
Transaction cost (λ)	0.10
Maintenance cost (ψ)	0.01
Down payment (δ)	0.30
Mortgage rate (r_M)	0.04
Risk-free rate (r_f)	0.02
Mean equity premium (μ)	0.04
Variance of stock return (σ_η^2)	0.157^2
Max. prop. of gross income req. to qual. for a mortgage (ω)	0.30
Autoregression parameter (ϕ)	1.0

In the calibration of the house price, we again follow Cocco's (2005) work. He estimates the annual growth rate of the log real house price as 1.59% and the standard deviation of the detrended log house price as 0.062. Nevertheless, he also points out that part of the increase is probably due to an improvement in the quality of housing, which cannot be accounted for using PSID data. Thus, a lower

[13] Considering the education profile of households, their incomes peek relative earlier than those with high school degree or even without high school degree.

value of 1% is taken into account in his simulation. As such, we use this value in the baseline case as well. Despite the fact that the aggregate labour income is correlated with cyclical fluctuations in house prices, we set it as zero in the benchmark case for simplicity.

The transaction cost caused by trading a house is set at 10%, which is approximately consistent with Smith et al.'s (1998) result of 8%–10%. We let the maintenance cost and down payment equal 1% and 30% of the value of the house, respectively. In general, mortgage loans require a higher rate than bonds, since they bear long-term interest rate risk, default risk and prepayment risk (Hu, 2005). In this model, we set the risk-free rate r_f as 2% per year and the annual mortgage rate r_M as 4%, which gives a 2% mortgage premium.[14] The excess return on stocks is the sum of two components: the mean equity premium μ, which we set as 4% following Fama and French (2002), and a stochastic element η_t, the annual standard deviation of which is parameterized to 0.157 based on Standard & Poor's 500 Index (Campbell and Viceira, 2002). As for the maximum proportion of gross income required to qualify for a conventional mortgage, we follow the construction of the affordability index in the US and fix it at 30%. This value is also consistent with the conventional identification of people with the housing affordability problem, which is based on whether they pay more than a certain fixed percentage (25% or 30%) of their income for housing (Stone, 1990). Table 1 summarizes the parameters used in the baseline case.

3.1 Baseline Simulation Results

Our simulations begin with a baseline case. First of all, we simulate the exogenous stochastic labour income, housing prices and stock return based on the setting up performed in the previous section. Using the Bellman equation, we compute the household's optimal consumption, housing stock, mortgage and portfolio choices. The special features of life-cycle models, such as the household's finite horizon and age-dependent labour incomes, indicate that the policy function does not converge to a steady state. Thus, this problem can only be solved by backwards induction. To calculate the optimal decisions for the next period, we need to update the (standardised) cash-on-hand and housing stock at the beginning of each period. Once the optimal matrix of decision rules is derived, we may use it to generate 50,000 simulated optimal paths from age 30 to age 80. Finally, the statistical averages are obtained by simulations.

Our prime purpose is to examine the impact of the housing affordability constraint upon an investor's optimal consumption, mortgage, portfolio choices and poverty status over a lifetime. In accordance with the baseline parameters, simulation results are calculated for two scenarios in the prime (standard) mortgage market: with (scenario 1) and without (scenario 2) the affordability constraint. Table 2 shows the summary of the evolutions of the mean shares for various assets relative to the total assets, average consumption, income, mortgage-to-total-assets ratio, LTI, LTV and fraction of investors lying below the average poverty line in all the age groups. The overall qualitative feature of the evolutions of the portfolio composition is roughly consistent with Cocco's (2005) results. First of all, in both scenarios, stock- and bond-holdings are much less important than real estate and human capital when they are measured relative to the total assets. After retirement, real estate and human capital together account for 98% and 92% of the total assets in both scenarios. Secondly, our results also show that human capital is the most important component of wealth at all ages. Early in life, the shares of human

[14] Most papers estimate that the average spread between the conventional mortgage rate and the Treasury bill rate is between 2.29% and 3.01% (Hu, 2005). Since we eliminate a number of potential risks in the model, we use a much lower annual mortgage rate premium of 2%.

capital in the total assets in scenarios 1 and 2 are as high as 69.9% and 74%. As the household ages, the fractions become smaller, and in the last age group, they are both around 42%.

Table 2: Shares of Assets and Variables by Age in the Prime Mortgage Market

Assets	Scenario 1: With Affordability Constraint				Scenario 2: Without Affordability Constraint			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Stocks	0.0035	0.0045	0.0063	0.0072	0.0396	0.0056	0.0229	0.0730
Bonds	0.0015	0.0016	0.0021	0.0028	0.0004	0.0001	0.0012	0.0020
Real Estate	0.2959	0.3268	0.3848	0.5565	0.2200	0.2007	0.2317	0.5100
Human Capital	0.6991	0.6671	0.6068	0.4335	0.7400	0.7936	0.7442	0.4150
Variables								
Consumption	25.27	25.90	24.11	21.00	58.68	62.52	49.23	22.01
Income	26.95	28.64	26.50	23.55	26.95	28.64	26.50	23.55
Mortgage-to-Total-Assets-Ratio	0.0129	0.0142	0.0153	0.0220	0.1126	0.1059	0.1188	0.0920
LTI	0.3489	0.3438	0.3361	0.3229	3.0100	2.1400	2.1301	1.6900
LTV	4.39%	4.35%	3.99%	4.3%	51%	53%	51%	24%
Average % of Households Living. Below Poverty Line	19.5%	19.75%	20.16%	19.3%	63.8%	18.8%	21.4%	33.2%

Note: In this table, financial asset is the sum of stocks, bonds and real estate. Total asset is the sum of the financial asset and human capital. Human capital is the present discounted value of future income with the annual discounted rate of 5% following Heaton and Lucas (2000). For each age group, we calculate the average stock, bonds, real estate and human capital proportions in the total assets. The poverty line is calculated as 60% of the contemporary median household income, which includes earnings, retirement income, dividends, interest and income from real estate (OECD and US Department of Commerce).

Another similarity is that we find increasing portfolio shares invested in stock over the lifetime as well. Stockholding rises from 0.35%(3.96%) to 0.72%(7.3%) with(without) consideration of the affordability constraint as the household ages. Nevertheless, there are some magnitude differences between our results and Cocco's (2005). Although Cocco (2005) predicts an increasing life-cycle share of stock investments, the predictions of stockholdings are much less successful in matching the observations and the predicted values are on average higher than those in the data. In the presence of a constraint mortgage, our model predicts much lower shares of stocks across different age groups. Close to retirement (age 50–65), the household's stockholdings on average are 0.63%, whereas Cocco predicts 9.9% in the same age group, which is much higher than the observation in the PSID data of 1.9%.[15] Despite our outcome at the baseline also showing mismatching with the empirical evidence,

[15] The mean portfolio shares by age in the PSID data are reported by Cocco (2005, Table 9). The average stockholding shares related to the total assets in the age groups <35, 35–50, 50–65 and >65 are 0.2%, 0.7%, 1.9% and 2.9%.

the further investigation into the sensitivity of the affordability constraint degrees in section 3.3 shows more promising results.[16]

Now we focus on the quantitative differences between scenario 1 and scenario 2. In the presence of the affordability constraint, mortgage debt is restrained not only by the household's labour income but also by the house value owned in each period. To avoid excess borrowing from banks, the household needs to assess its "ability" during the whole dynamic optimisation process. As a result, the household manages to maintain a fairly smooth consumption pattern and keep the mortgage debt burden spread more evenly across different age groups. More specifically, according to the simulation results in scenario 1, the optimal consumption tracks closely the change in labour income and exhibits a hump-shaped distribution with peaks in the household's 50s. After retirement, consumption decreases by 20% because of the reduction in labour income. When ignoring affordability, the consumption also has a hump shape, yet it is accompanied by much higher volatility throughout the lifetime: it falls from the peak value of 62.52 between the ages of 40–50 to 22.01 after retirement. Housing equity is well known as a mechanism to smooth consumption over time (Hurst and Stafford, 2004). According to the simulation, this mechanism functions well under the consideration of mortgage affordability. Likewise, the mortgage-to-total-assets ratio, LTI and LTV on average are much smaller in scenario 1 than in scenario 2. LTI and LTV are calculated standard, i.e., as a ratio of loan outstanding to current income and of loan outstanding to house value. Taking the LTV for example, if banks allow a household to increase the level of mortgage debt without any limitation, the LTV in early life is as high as 53% and declines slowly to 24% after retirement.[17] On the other hand, when the constraint is imposed on household lending, LTV in the early life drops significantly to 4.39%. Obviously, the imposition of the affordability constraint has a large impact on LTV and LTI. According to our model, the first possible reason is that the households simply choose not to adjust the size of their house (we have assumed two scenarios of budget constraint, "no adjust" and "adjust", see equation 15), because they are not allowed to take too much credit from banks. Lower probability of "adjust" leads to lower mortgage borrowing, which results in much smaller value of LTV and LTI. The second reason is that since we don't allow mortgage default in the model, under the constraint of the affordability, a rational utility-maximizing household would rather continue to borrow a very small amount of mortgage loan (even equal to zero) than defaulting. This also leads to much lower LTV and LTI compared to that in the absence of affordability constraint.

Furthermore, we are also interested in the poverty status of households in the numerical experiments. As a matter of fact, housing-caused poverty is also considered to be a measure of housing affordability. Instead of using a certain fixed percentage for the mortgage, housing-induced poverty is a sliding scale of affordability, which varies with income and household type.[18] According to Kutty (2005), housing-induced poverty occurs at very low levels of income for households conventionally not considered cost burdened, therefore, it can be seen as more sensitive to income than the cost-to-income measure.

[16] If the affordability constraint is relaxed (ω=0.5), the average stockholding in the age group 50–65 increases to 1.1%; see section 3.3, Table 4. Furthermore, we also report explicitly mortgage-to-financial-asset-ratio and mortgage-to-total-asset-ratio and average stock, bonds and real estate proportions in the financial assets and total assets in a presence and an absence of affordability constraint, respectively. For more details please see Appendix B, Table B4 and Table B5.

[17] Notice that the maximal LTV in the simulation is approximately equal to 70% given a 30% down payment. In addition, we assume that all the households enter the model with a certain level of housing assets. It is reasonable that the simulated LTV ratios without the affordability constraint are below 70%. On the other hand, based on the statistics for housing from the Survey of Consumer Finances (SCF) in the US, the average LTV is roughly 45.8% before retirement; see Yao and Zhang (2005b).

[18] Stone (1990, 2010) developed the notion of "shelter poverty" as a measure of the housing affordability problem. Another similar concept, "housing-induced poverty", is discussed by Kutty (2005).

To illustrate these, we need to quantify the poverty status of a household. There is a large literature on the measurement of poverty. Following the OECD and the US Department of Commerce, we first compute the average poverty lines for different age groups. The poverty line is calculated as 60% of the contemporary median household income including earnings, retirement income, dividends, interest and income from real estate. Then, via Monte Carlo simulations with 8000 iterations, we calculate the average percentage of households living below the corresponding poverty lines in each age group. As such, we have specified households' poverty status. According to our numerical experiments, the poverty status of households is also strongly influenced by the affordability constraint. On the one hand, without consideration of the affordability constraint, massive borrowing easily puts younger people into poverty: 63.8% of households in the 30–40 age group lie below the poverty line. Although the living situation improves as households age, some of them might suffer from poverty again after retirement. Meanwhile, compared with LTI and LTV, the poverty status is more sensitive to the income levels/age profiles of the households (compare the last three rows in Table 2, scenario 2). On the other hand, if people take their "ability to borrow" into consideration, the average percentage of households living below poverty lines drops strongly across the different age groups and fluctuates merely between 19.3% and 20.1%. This implies firstly, that the presence of affordability constraint reduces the sensitivity of households' poverty status to their income levels; secondly, that the housing-induced poverty measure recognizes that consumer choice plays a part in maintaining a high or low housing cost burden (Kutty, 2005). Against this background, it is important that policy makers need to take into account more than one affordability measure for deregulation and reform of housing and mortgage markets.[19]

3.2 Effect of the Affordability Constraint in the Subprime Mortgage Market

Another purpose of this paper is to assess quantitatively the influence of the housing affordability constraint when households confront different mortgage markets. Besides the prime mortgage market, there is a liberalised mortgage market known as the subprime mortgage market. This type of mortgage lending aims to increase the numbers of homeowners and the opportunities for those homeowners to create wealth. However, it comes at a price and is normally described as high-cost lending (Chomsisengphet and Pennington-Cross, 2006). As a result, the interest rates for subprime loans are substantially higher than those for prime loans. According to the report of the Freddie Mac Primary Mortgage Market Survey, the difference between the prime and the subprime market is two to three percentage points. Thus, we set the mortgage rate in the subprime market as 7% and the down payment ratio as 10% to match the features of the subprime market. On the other hand, the deregulation of the mortgage market has acted to increase the average debt burden while reducing the average down payment and it also seems to have contributed to the rise in house prices (Gan and Hill, 2009). As such, we also differentiate the prime and subprime mortgage markets by considering different magnitudes of the volatility of house prices and set the standard deviations of the detrended log house price in the prime and subprime market as 0.062 and 0.1145, respectively.[20]

Table 3 reports the primary results of the numerical experiment in the subprime mortgage market (scenario 4). For comparison purposes, we also publish part of the results in the prime market in this table and name it scenario 3. Firstly, we notice that the evolutions of the average optimal consumption have similar patterns in the two markets: they evolve smoothly and exhibit hump shapes with peaks in the households' 50s. We also notice that the housing affordability restriction influences the house-value-to-income ratio and the liquid-wealth-to-income ratio in the prime and subprime markets in

[19] It is also discussed by McCord et al. (2011).

[20] $\sigma_{p'}$ is estimated as 0.1145 by Campbell and Cocco (2015).

different ways. In the former, the two ratios are quasi-hump-shaped or hump-shaped and peak just before retirement. The house-value-to-income ratio starts at round 8 and reaches the peak value of 8.43 around retirement. Young households accumulate wealth slowly until retiring. After the age of 65, their wealth declines as they draw down the wealth accumulated during their working years to supplement their retirement income to pay for housing and non-housing goods expenses (Yao and Zhang, 2005b).

Table 3: Evolution of Variables by Age in the Prime and Subprime Mortgage Markets with the Affordability Constraint

Variables	**Scenario 3: Prime Mortgage Market** δ=30%, r_M=4%, $\sigma_{p'}$=0.062				**Scenario 4: Subprime Mortgage Market** δ=10%, r_M=7%, $\sigma_{p'}$=0.1145			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Consumption	25.29	25.93	24.14	21.03	24.68	26.43	24.55	23.34
House-Value-to-Income-Ratio	8.05	7.89	8.43	8.02	3.96	3.71	4.14	7.29
Liquid-Wealth-to-Income-Ratio	0.13	0.14	0.18	0.15	0.064	0.053	0.056	0.14
LTI	0.35	0.34	0.33	0.34	0.45	0.47	0.44	0.35
LTV	4.39%	4.35%	3.99%	4.3%	11.63%	10.58%	10.55%	7.08%

Note: This table reports the households' average optimal non-housing consumption, house-value-to-income ratio, liquid-wealth-to-income ratio, LTI and LTV based on 50,000 simulated optimal paths in the prime and subprime mortgage markets.

In the subprime mortgage market, the affordability constraint crowds out the advantage of the lower down-payment rate for young households. Compared with scenario 3, there is a significant drop in the house-value-to-income ratio in scenario 4. It begins at 3.96 and increases with fluctuations to 4.14. After retirement, it reaches the peak value of 7.29. In each period, households need to evaluate their capability to borrow before upgrading their housing investment. Since this investment becomes more expensive in the subprime market, younger households cannot keep their house value as high as that in the prime market. Not surprisingly, the deregulation of the mortgage market (subprime mortgage) has relaxed the borrowing constraint (a larger LTI in scenario 4 than in scenario 3), but the growing burden imposed by the high mortgage interest rate confines households' capability to accumulate house value; therefore, it causes a low house-value-to-income ratio but with an accompanying high LTI and LTV in the subprime market throughout the households' lifetime.

Finally, to examine the effectiveness of the affordability constraint in the subprime mortgage market, we also calculate the households' average optimal consumption, house-value-to-income ratio, LTI, LTV and housing-caused poverty in the same manner as in Table 2 and a brief summary is reported in Table B1 (see Appendix B). Unsurprisingly, the affordability constraint also has a strong impact on households' investment behaviour, and as a stabiliser, it is more effective in the subprime market than in the prime market. Taking the LTV for example, in scenario 6 Table B1 (without the affordability constraint), it exhibits a monotonous falling pattern with the peak value of 63.61% in households' 30s. Meanwhile, in scenario 5 (with the affordability constraint), a similar trend is found but with significantly lower values. It starts with the peak value of 11.63% and declines to 7.08% as the household ages. Likewise, the optimal consumption decreases on average by 70 percentage points before retirement, when compared with the prime market, in which the corresponding value is only 55 percentage points.

3.3 Sensitivity Analysis

To check the robustness of our findings and the effects of certain parameters of the model, we also conduct simulation experiments for alternative parameterizations. Specifically, we consider different degrees of the mortgage affordability constraint by changing the value of ω. In fact, the thresholds of the housing-cost-to-income ratio have increased in the US over time. The Housing and Community Development Act of 1974 set this ratio as 25%; the Omnibus Budget Reconciliation Act of System of 1981 increased it to 30%; in 1979 and 1983 Congress enacted laws establishing a system of preferences for housing assistance and one of the criteria was a housing cost burden in excess of 50% of income (Kutty, 2005).[21] As such, we consider the cases of lower (ω=0.25) and higher (ω=0.5) values of the maximum proportion of gross income required to qualify for a mortgage.

Table 4: Different Degrees of Affordability Constraint in the Prime Mortgage Market

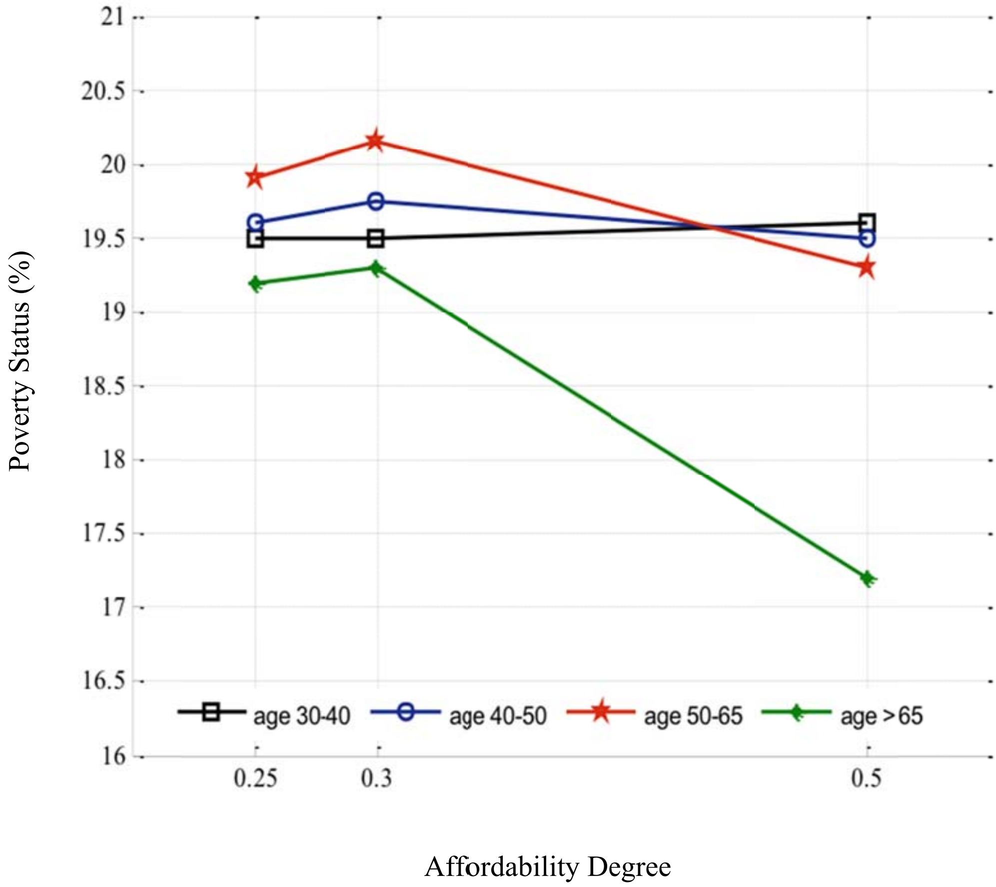
Assets	***ω*=0.25**				***ω*=0.5**			
	30-40	40-50	50-65	> 65	30-40	40-50	50-65	> 65
Stocks	0.0026	0.0038	0.0052	0.0050	0.0082	0.0088	0.0111	0.0120
Bonds	0.0014	0.0017	0.0019	0.0018	0.0012	0.0014	0.0019	0.0021
Real Estate	0.3071	0.3477	0.3998	0.5698	0.2598	0.2712	0.3282	0.5290
Human Capital	0.6887	0.6466	0.5929	0.4233	0.7308	0.7186	0.6588	0.4569
Variables								
Mortgage-to-Total-Assets-Ratio	0.0102	0.0105	0.0120	0.0150	0.0243	0.0279	0.0300	0.0336
LTI	0.2800	0.2634	0.2637	0.2328	0.63	0.62	0.61	0.48
LTV	3.3%	3.0%	2.9%	2.8%	10.0%	10.3%	9.2%	8%
Liquid-Wealth-to-Income-Ratio	0.11	0.14	0.16	0.1	0.25	0.23	0.26	0.19
Average % of Households Living. Below Poverty Line	19.5%	19.6%	19.9%	19.2%	19.65%	19.5%	19.3%	17.2%

Note: This table reports the average portfolio shares of various assets relative to total assets and the average optimal mortgage-to-total-assets ratio, LTI, LTV, liquid-wealth-to-income ratio and percentage of households living below the poverty line based on 50,000 simulated optimal paths in the prime mortgage market. The other parameters are held the same as in the baseline case.

[21] Nevertheless, the 30% of income standard is still widely used in the US. According to the 2006 American Community Survey (ACS), 37% of owners with mortgages and 16% of owners without mortgages spend 30% or more of their income on housing costs (Schwartz and Wilson, 2008).

Table 4 compares the mean portfolio shares, average optimal mortgage-to-total-assets ratio, LTI, LTV, liquid-wealth-to-income ratio and percentage of households living below the poverty line for these cases. The overall impression is that the portfolio shares in both cases evolve with a similar pattern to that in the baseline case. When the constraint becomes less strict (ω=0.50), a household has access to a higher loan such that it becomes more active in the equity market and accordingly more liquid assets are accumulated, resulting in a higher liquid-wealth-to-income ratio. Meanwhile, a less strict affordability constraint implies a heavier debt burden. Therefore, the mortgage-to-total-assets ratio, LTI and LTV are on average higher than those in the baseline case and in the case of ω=0.25.

Figure 3: Hump-shaped Relation between Affordability Degrees and Poverty Status



Note: Poverty status is calculated as the average percentage of households living below the corresponding poverty lines in different age groups.

One surprising result is that the average percentage of households living below the poverty line for all the age groups in the relaxed constraint (ω=0.50) case are approximately equal to or even smaller than those in the other cases, including the baseline case and the case with restrained affordability constraint (ω=0.25). This finding indicates that there is a non-linear (hump-shaped) relation between affordability degrees and poverty status, particular in the age groups above 40 (see Figure 3). One possible explanation is that relaxed constraint (higher ω) on credit allows homeowners aged above 40 easier to hoard more cash-on-hand to smooth their consumption and rebalance the financial portfolios. Therefore, they are able to acquire a higher return on investment from both equity market and mortgage market. Although the poverty status of younger group (age 30-40) is insensitive to the affordability degrees, the presence of affordability constraint prevents them from trapping in poverty.

Recall that if the constraint is ignored as discussed earlier (scenario 2 in Table 2), the heavy debt burden squeezes out the benefits obtained in the equity market, and younger homeowners will find it is difficult to maintain a quality of life above the poverty line. From a policy standpoint, this implies that in light of the age profile of households and feature of credit markets, the magnitudes of the borrowing constraint are needed to be carefully assessed in order to minimize the potential conflicts with other social policies fostering home ownership and credit availability.

Another important component of our model is the housing asset, and as such we examine the impact of the affordability constraint for different parameters related to housing as well, including the preference for housing θ, real log house price growth rate g and standard deviation of the detrended log house price $\sigma_{p'}$. When the importance of housing increases, due to the limited access to mortgage loans, households have to reduce their participation in the equity market (smaller stockholdings) in exchange for more investment in real estate. Households' average stockholding and bondholding decrease while their housing asset and their house-value-to-income ratio increase throughout their lifetime. On the other hand, without the limitation on borrowing, it is no longer necessary for households to quit the stock market. However, massive investment in both equity and housing markets easily brings them into severe poverty not only early in life but also after retirement (see Appendix B, Table B2).

Similar to this, a higher real log house price growth rate motivates people to invest more in real estate, but the affordability constraint as a stabiliser again weakens the boost in housing and stockholding. The housing asset is well known as a self-insurance instrument against future risk. If households expect higher house price risk, they have a much stronger incentive to invest in real estate, resulting in a lower level of stockholding at the same time. This mechanism of the housing asset is well documented by many researchers, such as Banks et al. (2012), Cocco (2005) and Han (2010). However, in the presence of the affordability constraint, households' investment behaviour is strictly restricted, making the hedging effect and crowding-out effect of house price risk less straightforward.

The fixed return to riskless bonds r_f also has an impact on our affordability constraint. For a given mortgage rate, the lower r_f is, the less strict the restriction is and vice versa. Accordingly, we set r_f at a lower (0.01) and a higher value (0.03), respectively, and the overall impression is that in the presence of affordability the patterns of the average portfolio composition, LTI, LTV, house-value-to-income ratio, liquid-value-to-income ratio, poverty status and non-housing goods consumed resemble those in the baseline case. However, there are also some quantitative differences that are worth noticing. For instance, the share of bondholding increases across all the age groups, since a higher r_f indicates a higher return on bonds. On the other hand, a higher r_f also implies a stricter affordability constraint. As such, in the case with a higher r_f, the average LTI and LTV drop and consequently the living quality improves (fewer households living below the poverty line) in all the age groups. In addition, the magnitude of these variations becomes much larger in the case without considering the affordability limitation.

Last but not least, we are also interested in the effect of the mortgage rate r_M in the model. In the baseline case, we assume that mortgage lenders require a positive spread between the mortgage rate and the risk-free rate for compensation, because of the potential risks of default and refinancing. We now examine the effects of a lower mortgage premium, given the risk-free rate. In particular, we set r_M as 2% and 6%, which give a 0% and 4% mortgage premium, in the prime and subprime mortgage markets, respectively. As we expected, with limited access to credit, the patterns of the optimal average portfolio composition, credit borrowing behaviour, poverty status and consumption do not change much compared with the baseline case in both mortgage markets (see Appendix B, Table B3).

The reason is that the lower mortgage rate has a dual effect on households' investment behaviour in the presence of the affordability constraint. On the one hand, the lower mortgage rate means "cheaper" loans, and as such households are willing to enlarge their housing stock. On the other hand, a small r_M implies a more restricted borrowing constraint (according to equation 14), which leads to less mortgage borrowing. Our simulation results show that the mortgage, LTI ratio and mortgage-to-total-assets ratio across different age groups decrease slightly in both markets when r_M decreases, indicating that the affordability effect overtakes the benefit of "cheaper" loans in our model. However, if mortgage lenders allow households to borrow loans beyond their ability, we reach different conclusions. Apparently, households have more incentive to invest in real estate at a lower mortgage rate/smaller mortgage premium. As a consequence, the average mortgage, LTI and mortgage-to-total-assets ratio rise throughout the lifetime. In early life, a mortgage borrowed from the prime market increases from 80 to 113.8, which gives an increment of the LTI from 3.01 to 4.25. After retirement, the LTI declines slightly but is still as high as 3.13. Meanwhile, in the subprime market, the LTV ratio increases from 63.61 to 67.72% early in life, when the mortgage rate r_M decreases from 7to 6%. For all the alternative scenarios discussed in this section, the overall impression is that our sensitivity analysis confirms the robustness of the findings in the baseline case. Further, it emphasizes that the affordability constraint, along with its degree, is an important determinant of households' investment behaviour and poverty status caused by housing.

4. Conclusion

Housing affordability issues have been documented increasingly in the recent research literature; nevertheless, most studies focus mainly on the empirical measurement and examination of the affordability problem of homeowners or renters. The collapse of the global financial markets in 2007 sends us a clear sign that careful research is necessary to assess the extent to which the mortgage market must be restructured to deal with these problems. As such, we attempt to investigate the mortgage affordability problem in a well-developed, calibrated, rational and dynamic life-cycle modelling framework. By incorporating explicitly the affordability constraint in the mortgage market, we focus primarily on households' "ability to borrow". Our simulation results indicate that the affordability constraint has a significant impact on households' optimal non-housing consumption, LTI, LTV, portfolio shares and poverty situation with age. Especially for young households that take out mortgage debt aggressively, the heavy burden imposed by housing investment significantly reduces the total spendable resources on hand, easily putting them into poverty, whereas when households assess their borrowing ability more carefully, they can maintain a fairly smooth consumption pattern and evenly spread mortgage burden. Furthermore, this mechanism works more effectively in the subprime market than in the prime market. Additionally, we conduct simulation experiments with alternative parameterizations and our sensitivity analysis confirms the robustness of the findings in the baseline case. One surprising result is that we find a non-linear relationship between affordability degrees and poverty status. After the age of 40, relaxing constraints (higher ω) on credit allow people to hoard more resource to smooth their consumption and rebalance the portfolios. As such, they could obtain benefits from both equity and housing markets. Although the average LTV increases slightly across all the age groups, it is not a threat to the living quality after their retirement. Meanwhile, this study also serves as scenarios tests intended to investigate the impact of borrower-based macroprudential policies on social policies aimed at improving poverty status and fostering home ownership and credit availability.

The previous works show that the dynamic life-cycle model of portfolio choice and housing offers a useful framework to study consumption behaviour and investment choices. The advantage of this type of modelling is that many realistic scenarios can be framed well and studied quantitatively, such as a rental market for housing services (Yao and Zhang, 2005a), a fixed stock market participation fee (Cocco, 2005), a refinancing charge, a default penalty (Yao and Zhang, 2005b) and mortgage default decisions under an adjustable/fixed rate, inflation and taxation (Campbell and Cocco, 2015). The present paper can be added to the list of these scenario studies. However, some limitations of the dynamic life-cycle model need to be noted as well. Although many of them have been acknowledged and discussed before, in terms of the affordability constraint we have several concerns. First, labour income is the foundation of the construction of the affordability constraint in our model. Since we consider a partial equilibrium model, income is naturally assumed to be an exogenous and independent stochastic process. However, the true mechanism is far more complex than the one that we have used. For instance, if the income is correlated positively with the house price, housing becomes riskier, because it is not as good a hedge against labour income risk. Households prefer liquid financial assets to housing assets (Cocco, 2005). However, if this correlation is neglected, we expect a positive relationship between income and housing stock. A further limitation is that we assume that households evaluate their borrowing/affording capacity using the same affordability degrees in each period (annually), which is highly unlikely in reality. Since mortgage payment is usually determined by a standard annuity formula, renegotiation is associated with high costs that could affect households' intertemporal decisions.

The model is able to illustrate the impact of the affordability constraint throughout the lifetime; noting the limitations mentioned above, we do not suggest that it should be rigorously applied to housing investment activities. Nevertheless, considering our numerical experiments as suggestive evidence, it is apparent that sound standards for affordability assessment are needed to reduce the mortgage payment problems and avoid the incidence of mortgage default. In a deeper sense, this is also propitious to contain systemic risks and the macroeconomic costs of financial instability. A possible extension of this paper is to consider more than one affordability assessment while updating the calibration of the model by using multi-country OECD data for a more comprehensive and comparative analysis.

Appendix A

Standardizing and Backward Induction:

We simplify the household's optimisation problem by standardising its continuous choice variables with the permanent component of labour income Y_t'. Let $c_t = C_t/Y_t'$ be the consumption-to-income ratio, $b_t = B_t/Y_t'$ the bonds-to-income ratio, $s_t = S_t/Y_t'$ the stocks-to-income ratio, $\alpha_t = S_t/(S_t + B_t)$ the proportion of liquid assets invested in stocks over the sum of bonds and stocks, $h_t = P_t H_t/Y_t'$ the house-value-to-income ratio and $m_t = M_t/Y_t'$ the mortgage-to-income ratio. By assuming the Cobb–Douglas utility function and proportional housing maintenance and transition costs, we ensure that the numeracy good consumption, housing service, mortgage loan and portfolio rules $\{c_t,\ h_t,\ m_t, \alpha_t\}$ are independent of the household's income level. Consequently, the relevant state variables for the household's problem can be written as $x_t = \{q_t,\ h_t'\}$, where $q_t = Q_t/Y_t'$ is the wealth-to-income ratio and $h_t' = P_t H_{t-1}/Y_t'$ is the household's beginning-of-period house-value-to-income ratio. Similarly, the standardized budget constraint, mortgage borrowing and affordability constraint are given as:

$$q_t = \begin{cases} c_t + b_t + s_t + \psi h_t' - m_t, & No\ Adjust \\ c_t + b_t + s_t + (1 + \psi + \delta) h_t - (1 - \lambda) h_t' - m_t, & Adjust \end{cases}$$

(A1)

where

$$0 \le m_t \le (1 - \ \delta) h_t, \tag{A2}$$

$$\frac{M_t}{Y_t} = \frac{m_t}{\exp(f(t, Z_t))\exp(\varepsilon_t)} \le \omega \frac{R_f - \left(\frac{1}{R_f}\right)^{T-1}}{R_M - \left(\frac{1}{R_M}\right)^{T-1}} \frac{(R_M - 1)}{(R_f - 1)}. \tag{A3}$$

Defining $v_t(a_t) = \frac{V_t(X_t)}{\left(\frac{Y_t'}{P_t^{\theta}}\right)^{1-\gamma}}$ as the value function after standardization, the Bellman equation can be written as follows:

$$v_t(x_t) = \max_{a_t} \left\{ \frac{\left({c_t}^{1-\theta} {h_t}^{\theta}\right)^{1-\gamma}}{1-\gamma} + \beta E_t[v_{t+1}(x_{t+1})] \right\}, \qquad \forall\ 1 \le t \le T, \tag{A4}$$

s.t.

$$c_t > 0, h_t > 0, m_t \ge 0, b_t \ge 0, s_t \ge 0\ ,$$

and equations (A1)–(A3).

The above problem can only be solved numerically using backwards induction (Judd, 1998). To compare our results with Cocco's (2005), we follow his way, using Gaussian quadrature methods to approximate the density functions of the exogenous random variables. For instance, the aggregate labour income process is approximated by a three-state transition probability matrix. Then, we discretize the state-space and the variables over which the choices are made with equally spaced grids. In the terminal period $(T+1)$, for each possible combination of the state variables, we can calculate the corresponding utility. According to the terminal condition, the utility function coincides with the value function, that is, $v_{T+1}(x_{T+1}) = u_{T+1}(x_{T+1})$. In every period t prior to $T+1$, we obtain firstly the utility associated with the different choice of control variables. Then, we calculate the value function at t, which is equal to the current utility plus the expected discounted continuation value associated with the choices made, and the given values of the state variables. Notice that the value function in each period t is only computed at discrete grid points, whereas it probably also needs to be evaluated between these points in the next period $t-1$ (backward), indicating that we have to compute them using interpolation and possibly extrapolation methods. This dynamic optimization process repeated recursively goes backwards until $t = 1$.

Construction of the Affordability Constraint:

In section 2.5, following Bourassa (1995), we define the affordability constraint as the ratio of the allowable total loan to the income in total allocated to the mortgage. In another word, we considered firstly the total loan and the total income across the life span in the model.

$$\sum_{n=1}^{T} \frac{E_1[M_n]}{R_M^{n-1}} \leq \sum_{n=1}^{T} \frac{\omega E_1[Y_n]}{R_f^{n-1}}, \tag{A5}$$

where ω is the maximum proportion of gross income required to qualify for a mortgage (affordability degree). Then, we have

$$E_1[M_1] + \frac{E_1[M_2]}{R_M} + \frac{E_1[M_3]}{R_M^2} + \cdots + \frac{E_1[M_T]}{R_M^{T-1}} \leq \omega[E_1[M_1] + \frac{E_1[M_2]}{R_M} + \frac{E_1[M_3]}{R_M^2} + \cdots + \frac{E_1[M_T]}{R_M^{T-1}}]. \tag{A6}$$

For simplicity, we assume the expected values of mortgage and income in each period are constant. Therefore,

$$\bar{M}\frac{1-\frac{1}{R_M^T}}{1-\frac{1}{R_M}} \leq \omega\bar{Y}\frac{1-\frac{1}{R_Y^T}}{1-\frac{1}{R_Y}}, \tag{A7}$$

$$\frac{\bar{M}}{\omega\bar{Y}} \leq \frac{R_f - \left(\frac{1}{R_f}\right)^{T-1}}{R_M - \left(\frac{1}{R_M}\right)^{T-1}} \frac{(R_M - 1)}{(R_f - 1)}. \tag{A8}$$

Through the rearrangement, we obtained the relation between $\bar{M}$ and $\omega\bar{Y}$. Notice that $\bar{M}$ and $\bar{Y}$ represent the mean of mortgage and income in each period. In the numerical experiment, we used this formula to evaluate the maximum mortgage that a household could borrow from the bank in period t, $1 \leq t \leq T$.

Appendix B

Table B1: Evolution of Variables by Age in the Subprime Mortgage Market

Variables	Scenario 5: With Affordability Constraint				Scenario 6: Without Affordability Constraint			
	30-40	40-50	50-65	> 65	30-40	40-50	50-65	> 65
Consumption	24.68	26.43	24.55	23.34	145.11	141.45	71.05	27.04
House-Value-to-Income-Ratio	3.96	3.71	4.14	7.29	24.61	20.26	10.45	7.63
LTI	0.4546	0.4745	0.4358	0.3533	16.16	11.40	4.61	1.46
LTV	11.63%	10.58%	10.55%	7.08%	63.61%	56.20%	40.0%	26.07%
Average % of Households Living. Below Poverty Line	19.3%	19.25%	19.17%	19.4%	49.06%	19.47%	21.75%	20.27%

Note: This table reports the households' average optimal non-housing consumption, house-value-to-income ratio, LTI, LTV and percentage of households living below the poverty line based on 50,000 simulated optimal paths in the subprime mortgage market.

Table B2: Robustness Analysis in the Prime Mortgage Market: Preference for Housing θ

	With Affordability Constraint							
	Baseline θ=0.1				**θ=0.3**			
Assets	30-40	40-50	50-65	> 65	30-40	40-50	50-65	> 65
Stocks	0.0035	0.0045	0.0062	0.0071	0.0028	0.0035	0.0044	0.0049
Bonds	0.0015	0.0016	0.0022	0.0028	0.0013	0.0014	0.0014	0.0010
Real Estate	0.2961	0.3272	0.3848	0.5565	0.5813	0.6267	0.6800	0.8000
Human Capital	0.6989	0.6667	0.6068	0.4226	0.4144	0.3682	0.3100	0.1932
Variables								
Consumption	25.28	25.91	24.11	21.00	20.01	20.52	18.81	16.87
House-Value-to-Income-Ratio	8.05	7.89	8.43	8.02	26.6	27.4	28.81	27.17
Average % of Households Living. Below Poverty Line	19.48%	19.75%	20.16%	19.3%	19.78%	20.01%	20.41%	19.43%
	Without Affordability Constraint							
	Baseline θ=0.1				**θ=0.3**			
Assets	30-40	40-50	50-65	> 65	30-40	40-50	50-65	> 65
Stocks	0.0390	0.0056	0.0229	0.0730	0.0450	0.0250	0.047	0.0370
Bonds	0.0004	0.0001	0.0012	0.0020	0.0012	0.0014	0.0013	0.0017
Real Estate	0.2214	0.2007	0.2317	0.5100	0.5971	0.6267	0.6782	0.8130
Human Capital	0.7400	0.7936	0.7442	0.4150	0.3561	0.3467	0.2732	0.1483
Variables								
Consumption	58.68	62.52	49.23	22.01	21.88	21.62	20.56	17.99
House-Value-to-Income-Ratio	5.86	4.06	4.14	8.08	31.77	29.08	33.34	34.33
Average % of Households Living. Below Poverty Line	63.8%	18.8%	21.4%	33.2%	54.72%	31.16%	52.82%	53%

Note: This table reports the average portfolio shares of various assets relative to the total assets and the average optimal house-value-to-income ratio and percentage of households living below the poverty line based on 50,000 simulated optimal paths in the prime mortgage market. The other parameters are set as the same values as in the baseline case.

Table B3: Robustness Analysis: Mortgage Rate r_M

	Panel A: Prime Mortgage Market							
	Baseline r_M=0.04				r_M=0.02			
	30-40	40-50	50-65	> 65	30-40	40-50	50-65	> 65
Assets	**With Affordability Constraint**							
Stocks	0.0035	0.0045	0.0063	0.0072	0.0025	0.0041	0.0052	0.0074
Bonds	0.0015	0.0016	0.0021	0.0028	0.0015	0.0017	0.0019	0.0031
Real Estate	0.2959	0.3268	0.3848	0.5565	0.3193	0.3588	0.4135	0.5842
Human Capital	0.6991	0.6670	0.6068	0.4335	0.6765	0.6352	0.5793	0.4076
Variables								
LTI	0.3489	0.3438	0.3361	0.3229	0.2350	0.2346	0.2203	0.2496
LTV	4.39%	4.35%	3.99%	4.3%	3%	2.6%	2.3%	2.8%
Average % of Households Living. Below Poverty Line	19.48%	19.75%	20.16%	19.3%	19.5%	19.77%	20.05%	19.4%
Variables	**Without Affordability Constraint**							
Real Estate	0.22	0.20	0.23	0.51	0.29	0.28	0.32	0.54
Mortgage	80.42	61.39	56.38	40.1	113.82	92.84	93.32	73.82
LTI	3.01	2.14	2.13	1.69	4.25	3.25	3.53	3.13
	Panel B: Subprime Mortgage Market							
	Baseline r_M=0.07				r_M=0.06			
	30-40	40-50	50-65	> 65	30-40	40-50	50-65	> 65
Assets	**With Affordability Constraint**							
Stocks	0.0009	0.0011	0.0012	0.0059	0.0018	0.0014	0.0018	0.0054
Bonds	0.0017	0.0016	0.0019	0.0077	0.0017	0.0017	0.0017	0.0073
Real Estate	0.1722	0.1874	0.2372	0.4941	0.1723	0.1877	0.2376	0.4945
Human Capital	0.8249	0.8098	0.7595	0.4921	0.8240	0.8091	0.7588	0.4925
Variables								
LTI	0.4546	0.4745	0.4358	0.3533	0.4393	0.4325	0.4148	0.3258
LTV	11.63%	10.58%	10.55%	7.08%	11.2%	11.6%	10.03%	6.6%
Average % of Households Living. Below Poverty Line	19.3%	19.25%	19.17%	19.4%	19.37%	19.27%	19.23%	19.38%
Variables	**Without Affordability Constraint**							
LTV	63.61%	56.20%	40.0%	26.07%	67.72%	54.2%	40.0%	27.2%

Note: This table reports the average portfolio shares of various assets relative to the total assets and the average optimal house-value-to-income ratio and percentage of households living below the poverty line based on 50,000 simulated optimal paths in both mortgage markets. The other parameters are set as the same values as in the baseline case.

Table B4: Shares of Assets and Variables by Age in the Prime Mortgage Market without Affordability Constraint

	Financial Assets				Total Assets			
Assets	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Stocks	0.1523	0.0272	0.0897	0.1247	0.0396	0.0056	0.0229	0.0730
Bonds	0.0016	0.0005	0.0046	0.0036	0.0004	0.0001	0.0012	0.0020
Real Estate	0.8461	0.9723	0.9057	0.8717	0.2200	0.2007	0.2317	0.5100
Financial Assets	1.000	1.000	1.000	1.000	0.2600	0.2064	0.2558	0.5850
Human Capital					0.7400	0.7936	0.7442	0.4150
Total Assets					1.000	1.000	1.000	1.000
Mortgage-to-Financial-Asset-Ratio	0.4330	0.5130	0.4644	0.1572				
Mortgage-to-Total-Asset-Ratio					0.1126	0.1059	0.1188	0.0920

Note: In this table, financial asset is the sum of stocks, bonds and real estate. Total asset is the sum of the financial asset and human capital. Human capital is the present discounted value of future income with the annual discounted rate of 5% following Heaton and Lucas (2000). Mortgage is reported relative to financial assets and total assets. For each age group, we calculate the average stock, bonds and real estate proportions in the financial assets and total assets. We also report the average human capital proportions in the total assets.

Table B5: Shares of Assets and Variables by Age in the Prime Mortgage Market with different Affordability Degrees

	Financial Assets				Total Assets			
Assets	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Panel A: Affordability Degree ω=0.3								
Stocks	0.0116	0.0135	0.0160	0.0127	0.0035	0.0045	0.0063	0.0072
Bonds	0.0050	0.0048	0.0053	0.0050	0.0015	0.0016	0.0021	0.0028
Real Estate	0.9834	0.9817	0.9787	0.9823	0.2959	0.3268	0.3848	0.5565
Financial Assets	1.000	1.000	1.000	1.000	0.3009	0.3329	0.3932	0.5665
Human Capital					0.6991	0.6671	0.6068	0.4335
Total Assets					1.000	1.000	1.000	1.000
Mortgage-to-Financial-Asset-Ratio	0.0428	0.0427	0.0389	0.0388				
Mortgage-to-Total-Asset-Ratio					0.0129	0.0142	0.0153	0.0220
Panel B: Affordability Degree ω=0.5								
Stocks	0.0304	0.0313	0.0325	0.0223	0.0082	0.0088	0.0111	0.0120
Bonds	0.0046	0.0050	0.0055	0.0037	0.0012	0.0014	0.0019	0.0021
Real Estate	0.9650	0.9637	0.9620	0.9740	0.2598	0.2712	0.3282	0.5290
Financial Assets	1.000	1.000	1.000	1.000	0.2692	0.2814	0.3412	0.5431
Human Capital					0.7308	0.7186	0.6588	0.4569
Total Assets					1.000	1.000	1.000	1.000
Mortgage-to-Financial-Asset-Ratio	0.6333	0.0991	0.0880	0.0610				
Mortgage-to-Total-Asset-Ratio					0.0243	0.0279	0.0300	0.0336

Note: In this table, financial asset is the sum of stocks, bonds and real estate. Total asset is the sum of the financial asset and human capital. Human capital is the present discounted value of future income with the annual discounted rate of 5% following Heaton and Lucas (2000). Mortgage is reported relative to financial assets and total assets. For each age group, we calculate the average stock, bonds and real estate proportions in the financial assets and total assets. We also report the average human capital proportions in the total assets. With or without affordability constraints, the relative level of mortgage in general is decreasing over time as we observe in the PSID data. Furthermore, our numerical results in the presence of affordability constraint with higher degree (ω=0.5) are more successful in matching the observations. For instance, during the earlier/later stage of life mortgage relative to financial assets is 0.6333/0.0610. Although these values are higher when compare to PSID data where the corresponding values are 0.503/0.060 (Cocco, 2005, Table 9). However, compare to what Cocco (2005) predicts (0.6830/0.5120) in his model, our results are much closer to the empirical data. Obviously, the degree of affordability constraint plays a big role in the dynamic life-cycle portfolio and housing model.

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