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A minimax regret model for leader-follower facility location problem

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Abstract Leader-follower facility location problem is usually consisting of a leader and a follower, in which the two competitors are going to locate new facilities sequentially. The traditional studies generally assume that the leader knows in advance the partial or full information about the follower's response when he makes decision. This assumption, however, may be invalid or impracticable in practice. In this paper, we consider that the leader needs to locate a predetermined number of new facilities without knowing any information about the follower's response. By separating scenarios on the follower's response with different number of new facilities, a minimax regret model is proposed for the leader so that his maximum possible loss can be minimized. Based on the structural characteristics of the proposed model, a set of solving procedures is provided with transforming the follower's nonlinear (fraction) programming model into a linear one. In the numerical experiments, the proposed model is compared to two other location model, deterministic model and risk model, and the efficiency of linearization in decreasing the computation

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time is verified. The results reveal that the proposed model is more applicable for the leader when there is no information about the number or probability distribution of the follower's new facilities.

Keywords Leader-follower facility location · Competition · Minimax regret model · Linearization

1 Introduction

The research on facility location problem begins with Weber's (1909) famous paper. After that, operations researchers, traffic engineers, economists, and others have discussed the location of diverse facilities, such as service facility (Farahani et al., 2019; Xia et al., 2015), medical facility (Zhang and Atkins, 2019), emergency facility (Sestayo and Castro, 2019), utility facility (Ahmadi et al., 2016) and so on. Hotelling (1929) analyzed the location choice and pricing decision of two competitors on a finite line with uniformly spread consumers, which differs from the classic facility location problem and is generally agreed as the first paper on competitive facility location problem. In the field of competitive facility location studies, the authors incorporate the fact that other facilities are already (or will be) in the market and that any new facility(ies) will have to compete with them for its (their) market share (Plastria, 2001).

For competitive facility location problem, the customer choice rule for patronizing facilities is of great concern. The two mainly used rules are deterministic utility rule and random utility rule. Deterministic utility rule means that customers only visit the facility which gives them the highest utility, for example they only visit the closest facility or the facility offering the cheapest product. For random utility rule, customers are assumed to distribute their demand with certain probability. Gravity-based rule proposed by Reilly (1931) and later used by Huff (1964) and Huff (1966) is the most widely used random utility rule, in which customer's probability to patronize a facility is proportional to the facility's attractiveness and inversely proportional to the distance between the customer and the facility. Some new customer choice rules are proposed in recent years. For example, Fernández et al. (2017) proposed multi-deterministic utility rule (also named partially binary rules) in their work, which assumes that customers neither visit only one facility nor distribute their demand to all facilities. They only visit the most attractive facility in each company, and distribute their demand among all these most attractive facilities according to the gravity-based rule. Kung and Liao (2018) explained that consumer's demand are affected by the number of facilities, with the increasing number of new facilities, the total demand will grow. In Fernández et al. (2019), a probabilistic customer's choice rule is proposed, in which customers only patronize those facilities for which they feel an attraction greater than or equal to a threshold value. In this paper, we take the classical gravity-based rule to deal with competitive facility location problem, which is the most widely used rule and the base of aforementioned new rules.

According to competition pattern, the competitive facility location can be classified into four categories (Kress and Pesch, 2012): (1) static competition, in which the competitors are fixed and the players know all information; (2) dynamic competition, in which players repeatedly reoptimize their locations; (3) simultaneous competition, in which two rational competitors make their decisions at the same time to reach Nash equilibrium; and (4) sequential competition, which includes two types of players, leaders who choose locations at given instants and followers who make their location decisions based on the past decisions of the leaders. The solution concept generally employed in sequential location problems is the Stackelberg equilibrium: assuming rational players, and the location of each player is determined by backward induction. In this paper, we focus on the sequential competition and call it as leader-follower facility location problem in the rest of this paper.

Hakimi (1983) first introduced the leader-follower issue in the competitive facility location problem. He formally introduced the terms $(r|p)$-centroid problem and $(r|X_p)$-medianoid problem with one leader and one follower locating p and r facilities, respectively. In a $(r|p)$-centroid problem, the leader will locate p new facilities with the belief that the follower will invest r new facilities later. The $(r|X_p)$-medianoid problem is to locate r new facilities for the follower in order to maximize her market share, knowing that the leader has located p new facilities. Furthermore, Hakimi has proven that the leader-follower problems in $(r|p)$-centroid and $(r|X_p)$-medianoid cases are N-P hard. Moore and Bard (1990) pointed out that the leader and the follower always conflict with each other when both of them aim to optimize their objective functions, which means if one competitor's market share grows, the other will decrease.

Most researchers assumed that the leader knows exactly how many new facilities will be opened by follower, such that the leader-follower problem can be formulated as deterministic models. Hakimi (1986) solved the deterministic leader-follower model by relaxing the condition of fixed demand in a network space. Serra and ReVelle (1994) proposed two heuristic algorithms for leader-follower model, in which the leader and the follower locate the same number of facilities and customers only choose to patronize the closest facility. Fischer (2002) formulated the model in discrete space with the assumption that accurate number of new facilities will be built. The leader and the follower want to decide the locations and also the price of product, and a heuristic algorithm is developed to solve the problem. Peréz and Pelegrín (2003) formulated a leader-follower model in a tree and assumed that only one facility is to be located by both of them, two algorithms are proposed to generate all optimal locations for the leader, then the entire set of Stackelberg solutions are formed. Sáiz et al. (2009) also assumed that the leader and the follower only locate one facility, but they formulated the model in a continuous space and proposed a branch-and-bound method to solve their problem. Drezner and Drezner (1998) developed three heuristic methods to deal with a similar model to Sáiz et al. (2009). Qi et al. (2017) introduced service distance limitations in the leader-follower issue with the assumption that both the leader and the follower plan to open certain number of facilities. Gentile et al. (2018) considered three pairs of objective functions for the leader and the follower with predetermined number of facilities for them and branch-and-cut algorithms are used to solve the proposed models. Some recent studies also considered how to decide service quality, radius of influence, product variety, routing and so on, but the number of follower's new facilities is still deterministic in these studies (Aboolian et al., 2007; Saidani et al., 2012; Wang and Ouyang, 2013; Drezner et al., 2015; Redondo et al., 2015; Lopes et al., 2016; Sedghi et al., 2017; Dilek et al., 2018).

As the leader acts ahead of the follower and they are in competition, it is reasonable to assume that the leader has no information on the follower's number of new facilities in advance. Ashtiani et al. (2013) built a robust model for the leader-follower problem with inaccurate number of follower's new facilities in a discrete space. They defined their model as risk model with known probability distribution, which made a trade-off between expect value and deviation of leader's market share. After reviewing the literature on this topic, this conclusion can be drawn that previous studies have modeled the leader's problem with respect to the assumption that the number of follower's new facilities are known or with known probability distribution. However, in a competitive market, the information of the number or the probability distribution of follower's new facilities can not be captured easily by leader. Most worsely, a wrong estimation on the probability distribution may lead to more loss for the leader. A realistic example of leader-follower problem is about retail store location. Jingdong (JD), as one of the leading companies in China, has invested a considerable amount of capital to retail industry. According to the CEO's interview of JD, more than 1000 convenience stores were opened per week in 2018, and they aim to open 1000 stores in China each day in 2019. At the same time, competitions are everywhere in the market. There are some local convenience store brands, who are the main competitors for JD in each province. For example, Everyday is a convenience store company in Shanxi province, which opens some new stores each day since 2018, and the information of Everyday's decision is unknown for JD.

The contributions of this paper include: (i) We propose a minimax regret model for the competitive facility location problem consisting of a leader and a follower, in which the leader has no idea in advance on the number or probability distribution of the follower's new facilities when he makes decision; (ii) Based on the structural characteristics of the proposed model, a set of solving procedures is provided with transforming the follower's nonlinear (fraction) programming model into a linear one; (iii) The numerical experiments show that the minimax regret model is brilliant when the number of follower's response is unknown, by which we can better control the maximum possible loss compared to deterministic model and risk model.

The rest of this paper is organized as follows: In Section 2, a minimax regret model for leader-follower problem is formulated. In Section 3, the model is linearized and the solving procedures are introduced. In Section 4, the computational experiments and analysis results are presented in detail. In Section 5, conclusions and future research directions are provided.

2 Problem description and formulation

We consider a facility location problem consists of two competitors, a leader and a follower. They have established N_l and N_f facilities in the market and now plan to open new facilities. The decision sequence is that the leader launches facilities first and the follower launches facilities later. Both the leader and the follower are assumed to be rational such that the follower will open her stores at the optimal locations after the leader makes his decisions. The notations in Table 1 are used to formulate the problem.

The leader is going to launch p new facilities in candidate locations while the follower responds to leader's action by opening some new facilities. Two competitors provide identical services and the demand is considered inelastic and is assumed to be concentrated at K demand points in the market. It is also assumed that the number of facilities in each candidate location is no more than one, i.e., facilities cannot overlap at the same point. Since there are already $N_l + N_f$ facilities, the rest of $M = K - (N_l + N_f)$ points are candidate locations for the leader and the follower.

Since the leader acts firstly to choose p locations, he would know nothing about the follower's decision information. That is, the leader knows nothing about the number and the probability distribution of the follower's new facilities, and which would bring risk to him. With follower's different decision, the leader's optimal decision would change. So unreasonable location would lead to the leader's loss of the market share, and he should consider how to avoid this part of risk. The leader has no idea on the exact number of the follower's new facilities or the probability distribution, but the maximum number of the follower's new facilities W is assumed known for the leader. We accordingly define W scenarios, and in scenario ω the follower will open p_ω new facilities. Actually the follower is assumed to open $1, 2, \cdots, p_W$ new facilities in each scenario. The leader's problem is where to locate his p new facilities facing W scenarios.

2.1 Attractiveness

Gravity-based model is a widely employed model in the competitive facility location problem. According to Huff rule (Huff, 1964; Huff, 1966), the facility attractiveness level for customers should be proportional to the quality and inversely to the squared distance between the customer and the facility. It is supposed that the quality level of all existing and new facilities and the distance between candidate location and demand point are predetermined.

Denote q_{nk} as the quality of existing facility n for demand point k and use l_{nk} to denote the distance between existing facility n and demand point k,

Table 1 List of notations

Parameters	
$\mathbb{N}_l$	the set of existing leader's facilities with index $n = 1, 2, \cdots, N_l$
$\mathbb{N}_f$	the set of existing follower's facilities with index $n = 1, 2, \cdots, N_f$
$\mathbb{N}$	$\mathbb{N}_l \cup \mathbb{N}_f$, i.e., the set of existing facilities with index $n = 1, 2, \cdots, N_l + N_f$
$\mathbb{W}$	the set of scenarios with index $\omega = 1, 2, \cdots, W$
$\mathbb{M}$	the set of candidate locations with index $m = 1, 2, \cdots, M$
$\mathbb{K}$	the set of demand points with index $k = 1, 2, \cdots, K$
$\mathbb{X}$	the set of leader's strategy with index $X = 1, 2, \cdots, C_M^p$
b_k	the buying power at demand point k with $k \in \mathbb{K}$
l_{nk}	the distance between existing facility n and demand point k with $n \in \mathbb{N}$ and $k \in \mathbb{K}$
l_{mk}	the distance between candidate location m and demand point k with $m \in \mathbb{M}$ and $k \in \mathbb{K}$
q_{nk}	the quality of existing facility n for demand point k with $n \in \mathbb{N}$ and $k \in \mathbb{K}$
q^l_{mk}	the quality of leader's new facility at location m for demand point k with $m \in \mathbb{M}$ and $k \in \mathbb{K}$
q^f_{mk}	the quality of follower's new facility at location m for demand point k with $m \in \mathbb{M}$ and $k \in \mathbb{K}$
p	the number of leader's new facility
p_ω	the number of follower's new facility in scenario ω with $\omega \in \mathbb{W}$
Decision Variables	
x_m	binary variable, which is equal to 1 if the leader decides to launch a new facility in candidate location m, 0 otherwise, $m \in \mathbb{M}$
y^ω_m	binary variable, which is equal to 1 if the follower decides to launch a new facility in candidate location m in scenario ω, 0 otherwise, $\omega \in \mathbb{W}$ and $m \in \mathbb{M}$

so the attractiveness level of the existing facility n for customers at demand point k is

$$a_{nk} = q_{nk} / \left(\varepsilon + {l_{nk}}^2\right), \quad \forall n \in \mathbb{N},\ k \in \mathbb{K}, \tag{1}$$

where ε is a small real number added to avoid denominator becoming 0 in case of the candidate location n overlapping with the demand point k. Similarly, the attractiveness levels of the leader's and the follower's new facilities for customers at demand point k are

$$a^l_{mk} = q^l_{mk} / \left(\varepsilon + {l_{mk}}^2\right), \quad \forall m \in \mathbb{M},\ k \in \mathbb{K}, \tag{2}$$

$$a^f_{mk} = q^f_{mk} / \left(\varepsilon + {l_{mk}}^2\right), \quad \forall m \in \mathbb{M},\ k \in \mathbb{K}. \tag{3}$$

Define two binary variables x_m and y^ω_m to represent the leader's and the follower's decision respectively. If x_m takes value one, then candidate location m is occupied by the leader. If $y^\omega_m = 1$, then candidate location m is occupied by the follower in scenario ω. Especially, when $x_m = y^\omega_m = 0$, then candidate location m is not occupied by any of them. Then the total attractiveness level of the leader's facilities for customers at demand point k is calculated by summing up the attractiveness levels of both existing and new facilities, that is,

$$L_k(X) = \sum_{n\in\mathbb{N}_l} a_{nk} + \sum_{m\in\mathbb{M}} a^l_{mk} x_m, \quad \forall k \in \mathbb{K}. \tag{4}$$

In equation (4), we have $X = \{x_1, x_2, \cdots, x_m\}$, which represents one leader's location strategy.

As the follower locate her facility after the leader, once the leader makes his decision X, the follower's optimal location $Y_\omega = \{y^\omega_1, y^\omega_2, \cdots, y^\omega_m\}$ in scenario ω can be obtained. Similarly, in each scenario ω, the total attractiveness level of the follower's facilities for customers at demand point k is

$$F^\omega_k(Y_\omega|X) = \sum_{n\in\mathbb{N}_f} a_{nk} + \sum_{m\in\mathbb{M}} a^f_{mk} y^\omega_m, \quad \forall k \in \mathbb{K},\ \omega \in \mathbb{W}, \tag{5}$$

where the first term is the attractiveness level of existing facilities and the second term is the attractiveness level of new facilities. In the equation, $Y_\omega|X$ is used to denote the follower's location decision in scenario ω with a leader's decision X.

As a result, the total attractiveness level of all facilities in the market for customers at demand point k in scenario ω is formulated as

$$T^\omega_k(X, Y_\omega) = \sum_{n\in\mathbb{N}} a_{nk} + \sum_{m\in\mathbb{M}} a^l_{mk} x_m + \sum_{m\in\mathbb{M}} a^f_{mk} y^\omega_m, \quad \forall k \in \mathbb{K},\ \omega \in \mathbb{W}. \tag{6}$$

The three terms on the right hand represent the attractiveness levels of all existing facilities, the leader's p new facilities and the follower's p_ω new facilities, respectively.

Remark 1 The qualities q^l_{mk} and q^f_{mk} are related to both location and demand point, which concides with the existing studies Ashtiani et al. (2013) and Qi et al. (2017). It is because the quality of a facility is affected by the location, besides, the preference of customers in different demand points are varied.

2.2 Constraints

The leader's behavior in our model is similar to the $(r|p)$-centroid problem, in which he will launch p new facilities after anticipating follower's response. The difference is on the follower's response. In $(r|p)$-centroid problem, the follower will surely invest r new facilities, but in our model, the number of her new facilities is unknown. Constraint (7) ensures that p new facilities are launched by leader, that is,

$$\sum_{m\in\mathbb{M}} x_m = p. \tag{7}$$

As mentioned before, the follower is likely to open $1, 2, \cdots, p_W$ new facilities associated with W scenarios. Then for each scenario $\omega \in \mathbb{W}$, constraint

(8) ensures that p_ω of the candidate locations are selected by the follower to launch new facilities, that is,

$$\sum_{m\in\mathbb{M}} y_m^\omega = p_\omega, \quad \forall \omega \in \mathbb{W}. \tag{8}$$

Note that the value and probability distribution on p_ω are unknown for leader in advance.

Constraint (9) limits the number of new facilities to be opened at each candidate location should be less than or equal to one, which restricts that the leader and the follower can not overlap their facilities in the same candidate location.

$$x_m + y_m^\omega \leq 1, \quad \forall m \in \mathbb{M},\ \omega \in \mathbb{W}. \tag{9}$$

Constraint (10) states the range of the decision variables x_m and y_m^ω, that is,

$$x_m \in \{0,1\},\ y_m^\omega \in \{0,1\}, \quad \forall m \in \mathbb{M},\ \omega \in \mathbb{W}. \tag{10}$$

2.3 Minmax regret model

Huff rule is used to describe customer's choice behavior. We set b_k as the buying power at demand point k, which is distributed to every facility with a certain probability. The probability is equal to the proportion of the facility's attractiveness level to all facilities' attractiveness levels. The demands captured by this facility from demand point k can be calculated by multiplying b_k with the probability. In this way, we can obtain the leader's market share by summing up the demands captured by all his facilities. Similarly, we can obtain the follower's market share. As we mentioned, the two competitors are rational, so they both try to look for the optimal locations for themselves. In each scenario ω, once the leader firstly launches p new facilities, the follower's problem is to decide her strategy Y_ω in the candidate locations. With a given leader's strategy $X = \{x_1, x_2, \cdots, x_m\}$, the follower's problem in each scenario $\omega \in \mathbb{W}$ can be formulated as the following model

$$\max \ \sum_{k\in\mathbb{K}} b_k \frac{F_k^\omega(Y_\omega|X)}{T_k^\omega(X, Y_\omega)} \tag{11}$$

$$\text{s. t.} \ \sum_{m\in\mathbb{M}} y_m^\omega = p_\omega, \tag{12}$$

$$x_m + y_m^\omega \leq 1, \quad \forall m \in \mathbb{M}, \tag{13}$$

$$y_m^\omega \in \{0,1\}, \quad \forall m \in \mathbb{M}. \tag{14}$$

The objective function (11) is to maximize the follower's market share collected by all her facilities, in which $F_k^\omega(Y_\omega|X)/T_k^\omega(X, Y_\omega)$ represents the aforementioned probability. Note that there are totally W scenarios and each scenario represents different number of the follower's new facilities. The constraints (12) - (14) have been explained in the last subsection.

By solving the follower's model, the optimal locations of the follower's new facilities and market share in each scenario can be obtained. Then we make decisions for the leader. In reality, it is reasonable for the leader to decide on the most likely scenario (in his judgement) and act accordingly. However, the consequence may be serious if that scenario does not materialize. The minimax regret criterion, which can control this kind of risk by minimizing the maximum possible loss under any of the likely scenarios, is used here to solve the leader's problem. For each scenario, the regret value for the leader associate with one strategy is the difference between the maximum possible market share captured at the optimal strategy and the market share captured at this strategy. By using $\mathbb{X}$ to denote the set of the leader's strategies and X to denote each strategy, we set $\pi_\omega(X)$ as the leader's market share associate with strategy X and scenario ω, and set π^*_ω which is the maximum value of $\pi_\omega(X)$, that is,

$$\pi^*_\omega = \max_{X\in\mathbb{X}} \pi_\omega(X) = \max_{X\in\mathbb{X}} \sum_{k\in\mathbb{K}} b_k \frac{L_k(X)}{T^\omega_k(X, Y^*_\omega(X))}. \tag{15}$$

In equation (15), we use $Y^*_\omega(X)$ to denote the follower's optimal location in scenario ω when facing the leader's strategy X.

According to the minimax regret criterion, the leader's problem is displayed as

$$\min_{X\in\mathbb{X}} \max_{\omega\in\mathbb{W}} [\pi^*_\omega - \pi_\omega(X)] \tag{16}$$

$$\text{s. t.} \sum_{m\in\mathbb{M}} x_m = p, \tag{17}$$

$$x_m \in \{0, 1\}, \quad \forall m \in \mathbb{M}. \tag{18}$$

The objective function (16) is to minimize the maximum regret value for the leader in all potential scenarios. Since the leader is going to launch p new facilities in M candidate locations, there are totally C^p_M potential strategies for him, then X takes values from 1 to C^p_M. Under this criterion, we can choose the strategy for the leader at which the maximum regret value for all leader's possible locations is minimized.

3 Model linearization and solution

The leader has a total of C^p_M possible strategies, and the follower has W s-cenarios. Given a leader's strategy and a scenario, the follower's model (11) - (14) becomes a deterministic one. Therefore, the leader-follower facility location problem can be settled by solving the follower's model (11) - (14) $C^p_M \times W$

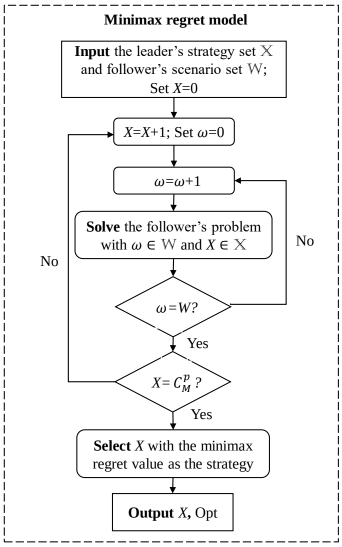


Fig. 1 The solving procedures of minimax regret model

times. After solving the follower's model per time, the follower's optimal locations and market share can be obtained, and then the leader's market share can be achieved correspondingly. When the leader's market shares under all these C_M^p strategies and W scenarios are obtained, the minimax criterion is applied to get the leader's optimal locations among all potential strategies. The solving procedures are depicted in Fig. 1.

In the solving procedures, the most crucial and difficult step is to solve the follower's model (11) - (14), which is essentially a nonlinear programming due to the blemish that the objective function (11) contains the fractional terms of decision variables. In the following, we devote to transform the follower's model (11) - (14) into a linear one, and then it can be efficiently solved by optimization solvers such as LINGO and CPLEX. According to Kochetov et al (2013), two new variables, z_k^ω and u_{mk}^ω, are introduced, which are defined as

$$z_k^\omega = \frac{1}{T_k^\omega}, \quad \forall k \in \mathbb{K}, \tag{19}$$

$$u_{mk}^\omega = z_k^\omega y_m^\omega, \quad \forall m \in \mathbb{M}, \quad k \in \mathbb{K}. \tag{20}$$

In equation (19), z_k^ω is basically the reciprocal of the total attractiveness of all facilities for a fixed k. In equation (20), u_{mk}^ω is formed by the multiplication of two variables, z_k^ω and y_m^ω. We utilize the property of 0-1 variable y_m^ω to transform the nonlinear follower's model to a linear one. Note that for the convenience of description, X, Y, and $Y_\omega^*(X)$ will not appear in the following formula. On this basis, we can derive the theorem.

Theorem 1 *For each $\omega \in \mathbb{W}$, the follower's model (11) - (14) is equivalent to the following linear form*

$$\max \sum_{k\in\mathbb{K}}\sum_{n\in\mathbb{N}_f} b_k a_{nk} z_k^\omega + \sum_{k\in\mathbb{K}}\sum_{m\in\mathbb{M}} b_k a_{mk}^f u_{mk}^\omega \tag{21}$$

$$\text{s. t.} \quad (12)-(14) \text{ and}$$

$$\sum_{n\in\mathbb{N}_f} b_k a_{nk} z_k^\omega + \sum_{m\in\mathbb{M}} b_k a_{mk}^f u_{mk}^\omega + b_k z_k^\omega L_k \le b_k, \quad \forall k \in \mathbb{K}, \tag{22}$$

$$0 \le u_{mk}^\omega \le y_m^\omega, \forall m \in \mathbb{M}, k \in \mathbb{K}, \tag{23}$$

$$u_{mk}^\omega \le z_k^\omega \le u_{mk}^\omega + S(1-y_m^\omega), \forall m \in \mathbb{M}, k \in \mathbb{K}. \tag{24}$$

In the above linear equivalent form, the decision variables are y_m^ω, z_k^ω and u_{mk}^ω. Note that S is a large enough constant in the constraint (24).

Proof For each $k \in \mathbb{K}$ and $\omega \in \mathbb{W}$, a constraint should be added to fulfill equation (19), that is,

$$z_k^\omega \le 1/T_k^\omega,$$

if both sides of the inequality are multiplied by $b_k T_k^\omega$, we have

$$b_k z_k^\omega T_k^\omega \le b_k,$$

because $T_k^\omega = F_k^\omega + L_k$, we get

$$b_k z_k^\omega F_k^\omega + b_k z_k^\omega L_k \le b_k.$$

The first term is equal to $\sum_{n\in\mathbb{N}_f} b_k a_{nk} z_k^\omega + \sum_{m\in\mathbb{M}} b_k a_{mk}^f u_{mk}^\omega$, then we get the linear constraint (22).

Constraints (23) and (24) are to fulfill equation (20), in which we utilize the property of 0-1 variable y_m^ω. When $y_m^\omega = 0$, for each $m \in \mathbb{M}$, $k \in \mathbb{K}$ and $\omega \in \mathbb{W}$, equation (20) turns to

$$u_{mk}^\omega = 0,$$

which is restricted by constraint (23). For constraint (24), as $y_m^\omega = 0$, it turns to

$$u_{mk}^\omega \le z_k^\omega \le u_{mk}^\omega + S,$$

which is always satisfies.

Similarly, when $y_m^\omega = 1$, for each $m \in \mathbb{M}$, $k \in \mathbb{K}$ and $\omega \in \mathbb{W}$, equation (20) turns to

$$u_{mk}^\omega = z_k^\omega,$$

which is restricted by constraint (24), while constraint (23) always satisfies. Note that when $y_m^\omega = 1$, $z_k^\omega \le y_m^\omega$ is a constantly established inequality because

$$z_k^\omega = \frac{1}{\sum\limits_{n\in\mathbb{N}} a_{nk} + \sum\limits_{m\in\mathbb{M}} a_{mk}^l x_m + \sum\limits_{m\in\mathbb{M}} a_{mk}^f y_m^\omega} \leq 1 = y_m^\omega,$$

then we have

$$0 \leq u_{mk}^\omega = z_k^\omega \leq y_m^\omega,$$

which means constricts (23) and (24) do not conflict with each other. Then no matter what the value of y_m^ω, equation (20) is always fulfilled. On this basis, constraints (22) - (24) ensure that equations (19) - (20) are valid.

As for the objective function (21), it is equivalent to (11) because

$$\begin{aligned}
&\sum_{k\in\mathbb{K}}\sum_{n\in\mathbb{N}_f} b_k a_{nk} z_k^\omega + \sum_{k\in\mathbb{K}}\sum_{m\in\mathbb{M}} b_k a_{mk}^f u_{mk}^\omega \\
&= \sum_{k\in\mathbb{K}} b_k \left(\sum_{n\in\mathbb{N}_f} a_{nk} + \sum_{m\in\mathbb{M}} a_{mk}^f y_m^\omega \right) z_k^\omega \\
&= \sum_{k\in\mathbb{K}} b_k F_k^\omega z_k^\omega \\
&= \sum_{k\in\mathbb{K}} b_k \frac{F_k^\omega}{T_k^\omega}.
\end{aligned}$$

In the above equation, both $\sum\limits_{k\in\mathbb{K}}\sum\limits_{n\in\mathbb{N}_f} b_k a_{nk} z_k^\omega + \sum\limits_{k\in\mathbb{K}}\sum\limits_{m\in\mathbb{M}} b_k a_{mk}^f u_{mk}^\omega$ and $\sum\limits_{k\in\mathbb{K}} b_k \frac{F_k^\omega}{T_k^\omega}$ represent the follower's market share, and we devote to maximize it.

As a result, follower's problem (21) - (24) is a linear equivalent form to (11) - (14). By the transformation, part optimal solution of the linear model (y_m^ω) is the optimal solution to (11) - (14). The proof is completed.

4 Numerical experiments

In this section, numerical experiments are conducted to verify the validity of the proposed model and test the efficiency of linearization. In order to further explain the necessity of our hypothesis about the number of follower's new facility is unknown or with unknown probability distribution, comparisons with two different location models are provided to show the advantages of the proposed model.

4.1 An illustration of minimax regert model

We consider an instance with 16 demand points and 5 existing facilities in the market, as shown in Fig. 2. Among the 5 existing facilities, 3 of them belong to the leader whose locations are (3,2), (4,3) and (1,4), and the other 2 of them

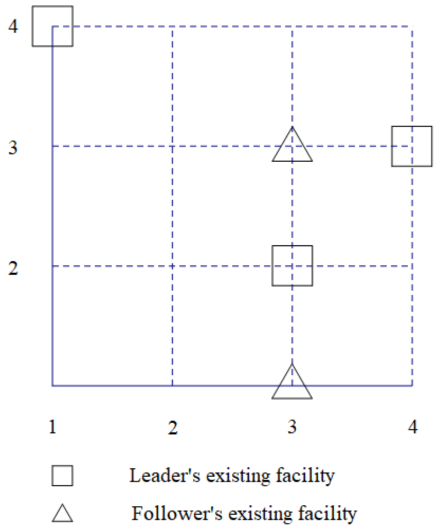


Fig. 2 The locations of the leader's and the follower's existing facilities

belong to the follower whose locations are (3,1) and (3,3). The leader aims to open 2 new facilities knowing that the follower will open some facilities after his action. However, he does not know the exact number of follower's facilities, but knows the maximum number is 4, i.e., the follower may open 1, 2, 3 or 4 new facilities. New facilities can not be opened in demand points that are already occupied by existing facilities. Therefore, the number of candidate locations for both leader's and follower's new facilities is 11. The buying power at each demand point is randomly generated from the range of 1-10 and we set the unit of buying power as million yuan. The buying power and coordinates of demand points are stated in Table 2. Quality values are also given randomly in the range of 1-5 for new and existing leader's and follower's facilities, which can be seen in Table 3.

Table 2 Coordinate and buying power of each demand point

Demand point	1	2	3	4	5	6	7	8
Coordinate	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)	(2,4)
Buying power	9	2	2	5	10	4	6	3
Demand point	9	10	11	12	13	14	15	16
Coordinate	(3,1)	(3,2)	(3,3)	(3,4)	(4,1)	(4,2)	(4,3)	(4,4)
Buying power	8	3	6	7	9	8	6	2

According to the solving procedures in Section 3, for each given strategy of the leader, we solve the follower's model firstly. Then the leader's market share can be obtained in each scenario. There are totally $C_{11}^2 = 55$ potential choices for the leader to choose 2 locations among 11 demand points as his strategy,

Table 3 The quality levels for different demand points

Quality	Leader				Follower		
	Existing			New	Existing		New
Demand point	(3,2)	(4,3)	(1,4)		(3,1)	(3,3)	
1	4	1	5	1	5	4	3
2	4	4	2	4	4	1	5
3	4	1	2	2	1	1	1
4	5	4	2	3	5	1	3
5	3	2	4	1	4	1	1
6	3	3	4	4	4	4	5
7	2	4	4	2	1	1	1
8	3	5	2	4	3	2	4
9	4	2	3	4	4	5	5
10	5	3	1	4	1	2	5
11	5	2	5	3	2	5	1
12	2	1	2	1	4	3	2
13	2	5	3	2	3	5	2
14	2	4	4	5	2	3	5
15	1	1	3	1	4	5	3
16	1	3	3	5	1	2	5

Table 4 Potential strategies of the leader

Strategy	Coordinates	Strategy	Coordinates	Strategy	Coordinates
1	(1,1),(2,1)	20	(4,1),(1,2)	39	(2,2),(3,4)
2	(1,1),(4,1)	21	(4,1),(2,2)	40	(2,2),(4,4)
3	(1,1),(1,2)	22	(4,1),(4,2)	41	(4,2),(1,3)
4	(1,1),(2,2)	23	(4,1),(1,3)	42	(4,2),(2,3)
5	(1,1),(4,2)	24	(4,1),(2,3)	43	(4,2),(2,4)
6	(1,1),(1,3)	25	(4,1),(2,4)	44	(4,2),(3,4)
7	(1,1),(2,3)	26	(4,1),(3,4)	45	(4,2),(4,4)
8	(1,1),(2,4)	27	(4,1),(4,4)	46	(1,3),(2,3)
9	(1,1),(3,4)	28	(1,2),(2,2)	47	(1,3),(2,4)
10	(1,1),(4,4)	29	(1,2),(4,2)	48	(1,3),(3,4)
11	(2,1),(4,1)	30	(1,2),(1,3)	49	(1,3),(4,4)
12	(2,1),(1,2)	31	(1,2),(2,3)	50	(2,3),(2,4)
13	(2,1),(2,2)	32	(1,2),(2,4)	51	(2,3),(3,4)
14	(2,1),(4,2)	33	(1,2),(3,4)	52	(2,3),(4,4)
15	(2,1),(1,3)	34	(1,2),(4,4)	53	(2,4),(3,4)
16	(2,1),(2,3)	35	(2,2),(4,2)	54	(2,4),(4,4)
17	(2,1),(2,4)	36	(2,2),(1,3)	55	(3,4),(4,4)
18	(2,1),(3,4)	37	(2,2),(2,3)		
19	(2,1),(4,4)	38	(2,2),(2,4)		

and scenario 1,2,3,4 in the following tables represent that the follower opens 1,2,3,4 facilities, respectively. The coordinates of strategy 1 - 55 for the leader are listed in Table 4. For example, if the leader chooses strategy 1, i.e., (1,1) and (2,1) to open two new facilities, the follower's problem is solved in candidate locations for different scenarios. For scenario 1 (the follower opens one new facility), the leader's maximum market share is 53.23 million yuan, while for scenario 2 (the follower opens two new facilities), the leader's maximum market share is 45.55 million yuan. Similarly, the market shares are 41.67 million yuan for scenario 3 (the follower opens three new facilities) and 38.60 million yuan for scenario 4 (the follower opens four new facilities). This is done for all 55 strategies and 4 scenarios in Table 5.

As observed in Table 5, the optimal strategy for the leader in scenarios 1 and 2 is 33, i.e., locations (1,2) and (3,4) are chosen, in which the market shares are 58.80 million yuan and 51.71 million yuan. Optimal strategy in scenarios 3 and 4 is 32, i.e., (1,2) and (2,4), in which the market shares are 48.13 million yuan and 45.39 million yuan. This reveals that with respect to the follower's different decisions, the leader's optimal strategy may be different. If the leader's optimal strategy among scenarios keeps unchanged, we can easily make decisions for the leader. However, the optimal strategy changes when the scenario is different, so we should deal with this issue brought by unknown number of follower's new facilities. Since the probability distribution of four scenarios is unknown for the leader, the minimax regret value rule is applied to make decision. The leader's optimal strategies of these 4 scenarios are displayed in Fig. 3.

The regret values for 55 leader's potential strategies in 4 scenarios are calculated and displayed in Table 6. In Table 6, each regret value means the loss of market share for one strategy when comparing with the maximum market share of that scenario. For example, strategy 33 is the location with the highest market share for leader in scenario 1, so the regret value of strategy 1 in scenario 1 is calculated as $58.80 - 53.23 = 5.57$, in which the two terms on the left hand represent the market shares of strategy 33 and strategy 1 in scenario 1, respectively. It is the same way to calculate the regret value of all the 55 leader's strategies in 4 scenarios. As a result, for each leader's strategy, we get 4 regret values correspond to 4 scenarios. For the leader's 4 regret values of each strategy, we take the maximum value and also list it in Table 6. Our purpose is to launch the leader's new facilities in the locations to ensure that the maximum regret value is minimized among all the 55 leader's strategies. According to the maximum regret value list in Table 6, the optimal strategy for leader is strategy 3, i.e., (1,1) and (1,2) because the maximum regret value in this location is only 0.99 million yuan, which is the lowest for all leader's potential strategies.

So, by choosing strategy 3, we can control leader's maximum loss to the lowest pitch. However, for the other 54 strategies, leader's maximum loss would be more than strategy 3 because the number of follower's new facilities is uncertain. The locations of strategy 3 are depicted in Fig. 4.

4.2 Comparison

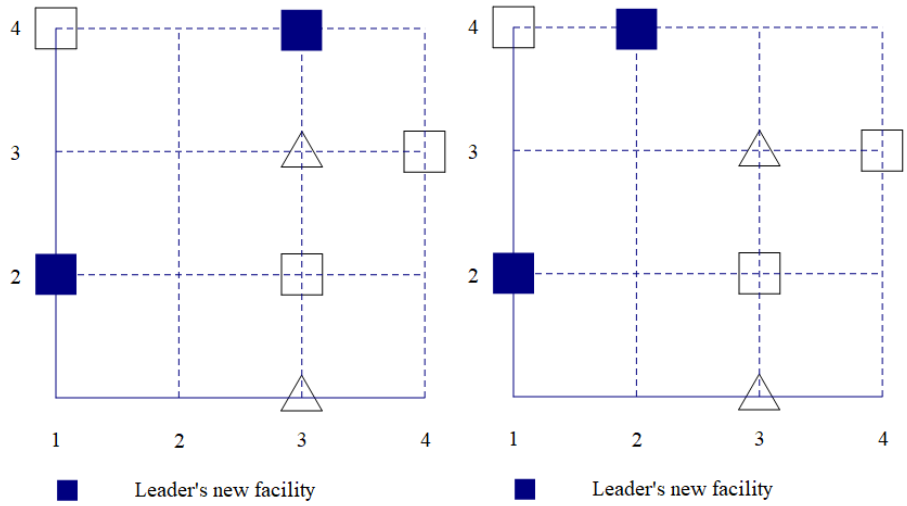
This paper studies the problem that the leader neither knows the number nor the probability distribution of the follower in the process of location, and propose the minimax regret model to control the leader's possible loss. In the previous literature, the deterministic model and the risk model have been widely studied in location problems. In the following, the minimax regret mod-

Table 5 Market share of the leader

Strategy	Scenario 1	Scenario 2	Scenario 3	Scenario 4
1	53.23	45.55	41.67	38.60
2	54.40	46.81	43.10	40.28
3	57.81	51.50	48.05	45.18
4	55.21	47.44	43.31	40.26
5	53.10	45.47	41.73	38.78
6	56.92	49.37	46.10	43.54
7	55.51	47.52	43.00	39.92
8	55.36	50.01	46.65	44.17
9	57.24	49.11	45.20	42.14
10	52.73	45.03	41.30	38.33
11	50.07	42.41	38.60	35.55
12	55.34	48.83	43.32	40.22
13	50.89	43.04	38.80	35.56
14	48.52	41.12	37.29	37.29
15	52.87	45.25	41.63	38.55
16	51.30	43.24	43.24	35.44
17	51.17	45.70	45.70	39.21
18	53.02	44.83	40.82	37.70
19	48.52	40.75	36.92	33.87
20	56.11	49.66	44.58	41.74
21	51.69	43.92	39.85	37.01
22	48.15	40.55	36.96	34.53
23	53.25	45.70	42.64	39.88
24	51.65	43.65	39.17	36.30
25	51.31	46.04	42.92	40.43
26	53.04	44.93	41.16	38.33
27	48.44	40.78	37.20	34.47
28	56.10	49.51	44.29	41.25
29	54.75	48.26	43.18	40.24
30	57.05	50.86	47.47	44.62
31	56.42	49.36	44.18	41.10
32	57.09	51.52	**48.13**	**45.39**
33	**58.80**	**51.71**	46.60	43.53
34	54.37	47.79	42.73	39.74
35	50.33	42.54	38.45	35.44
36	53.72	46.10	42.80	39.79
37	52.00	43.96	39.24	36.13
38	52.26	46.71	43.32	40.58
39	54.28	46.11	41.89	38.80
40	50.00	42.15	38.07	35.06
41	51.83	44.27	41.14	38.39
42	50.21	42.21	37.72	34.72
43	49.85	44.58	41.44	38.78
44	51.56	43.45	39.66	36.69
45	46.76	39.33	35.72	32.79
46	52.77	45.09	41.78	38.94
47	52.98	49.24	46.03	43.46
48	55.42	47.61	44.32	41.57
49	51.31	43.73	40.53	37.82
50	51.14	45.44	42.07	39.50
51	53.49	45.44	40.85	37.75
52	49.64	41.66	37.18	34.16
53	52.55	47.23	43.94	41.52
54	49.10	43.85	40.62	38.15
55	50.42	42.34	38.56	35.56

Table 6 Regret value of the leader

Strategy	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Maximum
1	5.57	6.15	6.46	6.78	6.78
2	4.40	4.89	5.03	5.10	5.10
3	0.99	0.21	0.08	0.20	**0.99**
4	3.59	4.27	4.82	5.13	5.13
5	5.70	6.24	6.39	6.61	6.61
6	1.88	2.34	2.03	1.84	2.34
7	3.29	4.19	5.13	5.47	5.47
8	3.44	1.69	1.48	1.22	3.44
9	1.56	2.60	2.92	3.24	3.24
10	6.07	6.68	6.83	7.06	7.06
11	8.73	9.30	9.53	9.83	9.83
12	3.46	2.88	4.81	5.16	5.16
13	7.91	8.67	9.32	9.83	9.83
14	10.28	10.59	10.84	8.10	10.84
15	5.93	6.46	6.50	6.84	6.84
16	7.50	8.47	4.89	9.94	9.94
17	7.63	6.00	2.42	6.18	7.63
18	5.78	6.88	7.31	7.68	7.68
19	10.28	10.96	11.21	11.51	11.51
20	2.69	2.05	3.55	3.65	3.65
21	7.12	7.79	8.28	8.38	8.38
22	10.65	11.16	11.17	10.86	11.17
23	5.55	6.01	5.48	5.51	6.01
24	7.15	8.06	8.96	9.08	9.08
25	7.49	5.66	5.20	4.96	7.49
26	5.76	6.78	6.97	7.06	7.06
27	10.36	10.93	10.93	10.92	10.93
28	2.70	2.19	3.83	4.14	4.14
29	4.05	3.45	4.95	5.14	5.14
30	1.75	0.85	0.66	0.77	1.75
31	2.38	2.35	3.94	4.29	4.29
32	1.71	0.19	**0.00**	**0.00**	1.71
33	**0.00**	**0.00**	1.52	1.86	1.86
34	4.43	3.92	5.40	5.64	5.64
35	8.47	9.17	9.68	9.94	9.94
36	5.08	5.61	5.33	5.60	5.61
37	6.80	7.75	8.89	9.26	9.26
38	6.54	5.00	4.81	4.81	6.54
39	4.52	5.59	6.24	6.59	6.59
40	8.80	9.56	10.06	10.32	10.32
41	6.97	7.43	6.99	7.00	7.43
42	8.59	9.50	10.41	10.67	10.67
43	8.95	7.13	6.68	6.60	8.95
44	7.24	8.25	8.47	8.70	8.70
45	12.04	12.38	12.41	12.60	12.60
46	6.03	6.62	6.35	6.45	6.62
47	5.82	2.47	2.09	1.93	5.82
48	3.38	4.09	3.81	3.82	4.09
49	7.49	7.98	7.60	7.57	7.98
50	7.66	6.27	6.06	5.89	7.66
51	5.31	6.27	7.28	7.64	7.64
52	9.16	10.04	10.95	11.23	11.23
53	6.25	4.47	4.19	3.87	6.25
54	9.71	7.86	7.51	7.24	9.71
55	8.38	9.37	9.57	9.82	9.82



(a) Scenarios 1 and 2 (b) Scenarios 3 and 4

Fig. 3 Optimal locations for the leader in different scenarios

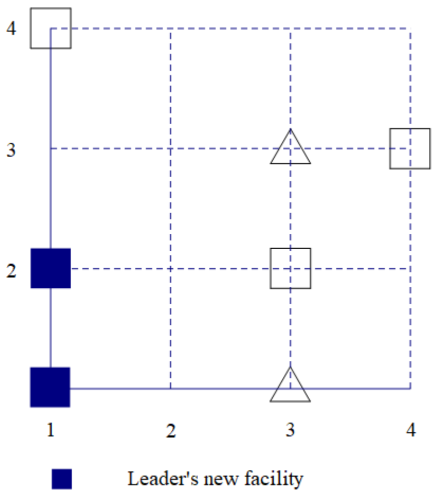


Fig. 4 The optimal locations for the leader in minimax regret model

el is compared with the deterministic model and the risk model to highlight the advantage of the minimax regret model and the serious consequences of blindly using the deterministic model and the risk model in the uncertain environment this paper concerns. If the leader enterprise blindly uses these two previous models to make the location decision, severe consequences would happen, i.e., the loss of market share arising from using wrong estimation on the number/distribution of followers new facilities.

4.2.1 Comparison with deterministic model

Starting with Hakimi's (1983) famous paper on leader-follower problem, most researchers considered accurate number of follower's new facilities, (See Sáiz et al., 2009; Kochetov et al., 2013; Gentile et al., 2018). For simplicity, we name this kind model as deterministic model, in which the leader is going to open p new facilities and he knows r new facilities will be opened by the follower.

This subsection is developed to compare the deterministic model and the minimax regret model. As we assumed, the leader has no idea on the follower's number of new facilities. If the leader still acts as a definite number of follower's new facilities, he is more likely to undertake the greater loss. And due to this, we make the comparison to show this part of loss. As shown in the numerical experiments, if the follower only open one new facility, strategy 33 is obviously optimal for the leader. However, in a competitive market, this kind of information can not be captured by the leader in advance, i.e., the number of follower's new facilities is unknown. If the leader only depends on an accurate number of follower's new facilities to launch new facilities, he would undertake greater loss. For example, the leader believes that the follower will only open one new facility and acts accordingly, he would choose strategy 33. But the follower launches 3 new facilities actually after leader's action, then the optimal strategy should be 32. As a result, leader's loss is 1.52 million yuan. However, if the minimax regret model is applied here to deal with the aforementioned situation, the loss is only 0.08 million yuan by selecting strategy 3.

It can be observed that no matter in which scenario, strategy 3 is not optimal solution for the leader, so even if the leader opens new facilities depends on the most likely scenario, he will never choose strategy 3 as its final decision. It is more likely for him to choose strategy 32 or 33 according to his judgement, because for scenarios 1 and 2, strategy 33 is optimal and for scenarios 3 and 4, strategy 32 is optimal. However, in the minimax regret model, strategy 3 is optimal, at which the maximum loss is only 0.99 million yuan. By contrast, if the leader choose strategy 32 or 33, which are the optimal strategies for deterministic model, the maximum loss can reach to 1.71 million yuan and 1.86 million yuan respectively. The results reveal that if we can not know accurately the follower's response, minimax regret value model can control the possible loss when comparing to deterministic model. The maximum loss and coordinates of strategy 3, 32 and 33 are recorded in Table 7.

Table 7 Comparison with deterministic model

Strategy	Coordinates	Optimal in which scenario	Maximum regret value
3	(1,1) and (1,2)	-	0.99
32	(1,2) and (2,4)	3 and 4	1.71
33	(1,2) and (3,4)	1 and 2	1.86

4.2.2 Comparison with risk model

In Ashtiani et al. (2013), the follower is assumed to open 1, 2,..., p_W new facilities with the probability of P_1, $P_2 ... P_W$. The leader's objective function is formulated as 'Expected Value - $\lambda \cdot$ Variance'

$$\max \sum_{\omega \in \mathbb{W}} \sum_{k \in \mathbb{K}} P_\omega b_k \frac{L_k}{T_k^\omega} - \lambda \sum_{\omega \in \mathbb{W}} P_\omega \left(\sum_{k \in \mathbb{K}} b_k \frac{L_k}{T_k^\omega} - P_\omega \sum_{k \in \mathbb{K}} b_k \frac{L_k}{T_k^\omega} \right)^2 .$$

This objective function maximizes the expected value of leader's market share and minimizes the deviation degree between the expected value and the scenarios' optimal solutions simultaneously. In the objective function, λ is a weight coefficient which measures the importance of variance. The rest of notations have the same meaning with our proposed minimax regret model.

It is a robust optimization for the leader, but its results may be affected both by the value of λ and the probability distribution of $P_1, P_2, \cdots, P_W$. To verify the possibility of the mentioned events, we test risk model with different values of λ, taking the value of 1, 0.8, 0.6, 0.4, 0.2 and 0 in sequence. Then for each λ, we generate 10000 probability distributions randomly. As shown in Fig. 5, leader is likely to choose strategy 3, 17, 32, 33 and 47 in risk model with the change of λ and probability distribution, which reveals that the sensitivity of risk model is at a high level. In practice, both λ and probability distribution are decided by leader's experience, so the values may be too subjective, and thus leading to unreasonable location. For example, when $\lambda = 1$, for 8296 probability distributions, the leader chooses strategy 47 to launch new facilities. For 927 probability distributions, the leader chooses strategy 32 and for 148 of them, the leader chooses strategy 17. Because the variance of leader's market share in different scenarios for strategy 47 is very small comparing to other strategies, although the market share of this strategy is not high in each scenario, it still be chosen as the optimal strategy in most cases. The market share of strategy 17 is even worse. The situations are analogous when $\lambda = 0.8$ and $\lambda = 0.6$.

As we assumed, the leader also has no idea on the probability distribution of the number of follower's new facilities. If the leader still acts as knowing the probability distribution, he is also more likely to undertake the greater loss. So we make the comparison to show this part of loss. For comparing risk model and minimax regret model, we calculate the weighted average of maximum loss associated with each λ, where the probability of choosing one strategy is denoted by frequency. For example, when $\lambda = 1$, the leader chooses strategy 47 with the probability of 82.96% and the maximum regret value is 5.82 million yuan. Similarly, the probability and maximum regret value for strategy 32 are 9.27% and 1.71 million yuan, for strategy 17 are 1.48% and 7.63 million yuan. Then the weighted average of possible maximum loss for $\lambda = 1$ is: $\overline{\text{Loss}} = 82.96\% * 5.82 + 9.27\% * 1.71 + 1.48\% * 7.63 = 5.10$ million yuan. The values of $\overline{\text{Loss}}$ for each λ are displayed in Table 8. It is obvious

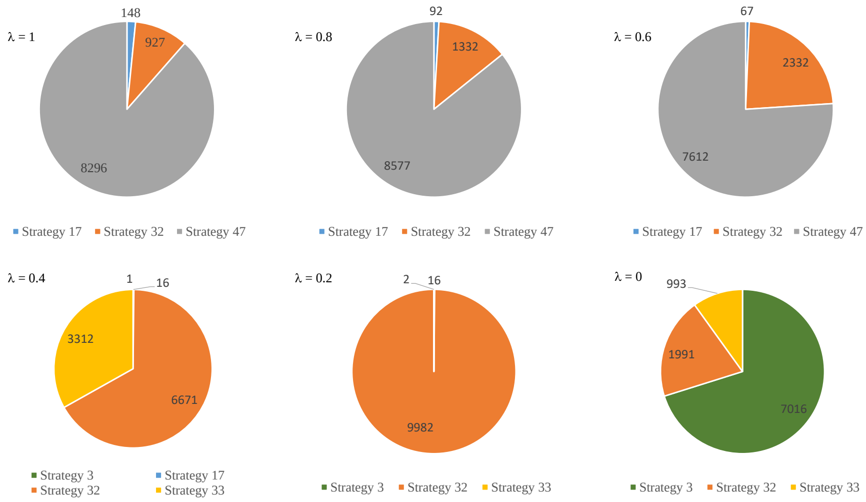


Fig. 5 The results of risk model with different λ

that $\overline{\text{Loss}}$ for all λ in risk model are higher than the possible maximum loss in minimax regret model. The maximum loss is only 0.99 million yuan when minimax regret model is applied, but the values of $\overline{\text{Loss}}$ in risk model are 5.10 million yuan, 5.29 million yuan, 4.88 million yuan, 3.08 million yuan, 1.71 million yuan and 1.07 million yuan, respectively. Even if that we do not consider the variance, the minimax regret model can still reduce the regret value from 1.07 million yuan to 0.99 million yuan, and the reduction is due to the wrong estimation on probability distributions.

All in all, when minimax regret value criterion is applied in leader's decision, more stable and less risk locations can be obtained. At the same time, there is no need to assume that the information of the number or the probability distribution of follower's new facilities is known by the leader in advance, which may be more practical in reality.

Table 8 Comparison with risk model

Risk model	λ=1	λ=0.8	λ=0.6	λ=0.4	λ=0.2	λ=0	Minimax regret model
$\overline{\text{Loss}}$	5.10	5.29	4.88	3.08	1.71	1.07	0.99

4.3 Test the efficiency of linearization

In the solving procedures, we transform the follower's nonlinear model (11) - (14) into a linear one. In this subsection, we use ten instances of different scales to test the efficiency of linearization. For each instance, the numbers of follower's new facilities, candidate locations and existing facilities are varied. The other required parameters are extracted the same way as stated before. The nonlinear models are solved by LINGO, while the linear models are solved by CPLEX. The results are recorded in Table 9. Note that '-' in Table 9 means the model can not be solved in 1000 minutes.

Table 9 Computation time for different sizes of the follower's model

p_ω	K	N_l	N_f	Market share (million yuan)		CPU time (min)	
				Nonlinear	Linear	Nonlinear	Linear
2	81	6	4	218.69	218.69	10.23	0.75
	100	6	4	304.88	304.88	25.02	1.74
3	81	6	4	246.19	246.19	120.67	11.58
	100	6	4	338.52	338.52	390.78	43.88
4	25	3	2	95.97	95.97	0.33	0.02
	36	3	2	124.72	124.72	4.26	0.22
	49	3	2	177	177	43.35	3.05
	64	3	2	241.90	241.90	340.35	28.51
	81	6	4	-	269.69	-	75.16
	100	6	4	-	363.51	-	685.42

In Table 9, it is obvious that for first eight instances, the follower's market shares are totally the same for either the linear model or the nonlinear model, but the computation time of the linear model is significantly decreased compared to that of the nonlinear model. For example, for the first instance with $p_W = 2$, $K = 81$, $N_l = 6$ and $N_f = 4$, the nonlinear model needs 10.23 minutes to be solved, while the linear model only needs 0.75 minutes to be solved. For the last two instances, the linear models can be optimally solved within acceptable time, but the nonlinear models would not obtain the optimal solution in up to 1000 minutes.

5 Conclusion and future research

This paper studies the leader-follower facility location problem. The main contribution of this paper is the formulation of a minimax regret model to control the leader's possible loss of location decisions without knowing any information about the follower's response. In the solving procedures, we transform the follower's model from a nonlinear (fraction) programming to a linear one by introducing new variables and constraints. Numerical experiments and comparisons are provided to verify the validity and advantages of the proposed model, and the efficiency of linearization is tested by using ten instances of different scales. The results reveal that compared to deterministic model and risk model, the proposed model is more applicable for the leader when there is no information about the number or probability distribution of the follower's new facilities. The nonlinear model is time-consuming or even unsolvable in up to 1000 minutes, while the linear model can significantly decrease the computation time.

For the future research, we can consider the take-out shops. Goods can be delivered so the delivery cost rather than the distance would be one influence factor of the facility's attractiveness. Also, with the development of delivery industry, goods are able to be delivered to some faraway customers, so it is necessary to develop some efficient meta-heuristic algorithms to solve large-scale problems. In addition, we can also consider the elastic demand for each demand point. For example, the buying power of demand points will rise with the increasing of new facilities, then the market situation for both leader and follower will be more complex.

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Conflict of interest

The authors declare that they have no conflict of interest.

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